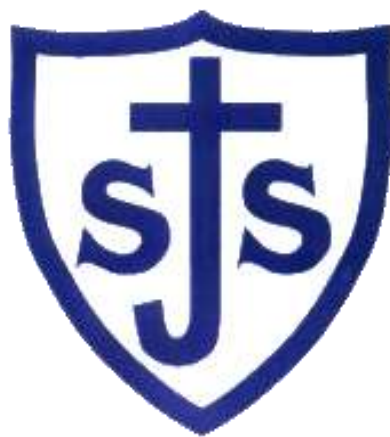


**ST JOSEPH'S CATHOLIC PRIMARY SCHOOL
CRAYFORD**



MATHS AND CALCULATION POLICY

Learning and growing together with Christ

Updated:	Spring 2025
To be Reviewed:	Spring 2028
Approved by:	Curriculum Committee

At St Joseph's, we are intent on maximising children's understanding and accuracy in applying mathematical concepts, knowledge, skills and thinking strategies to be successful in our modern world. We aim to achieve this through direct teaching, exploration, clarification, practise and application over time.

It is our intent that children develop a strong number sense so that as their confidence grows, they will be able to look for patterns, manipulate numbers, calculate mentally, use logical reasoning, suggest solutions and enjoy playing with and exploring numbers.

All pupils should:

- Become **fluent** in the fundamentals of mathematics, including through varied and frequent practice with increasingly complex problems over time, so that they develop conceptual understanding and the ability to recall and apply knowledge rapidly and accurately.
- **Reason mathematically** by following a line of enquiry, conjecturing relationships and generalisations, and developing an argument, justification or proof using mathematical language
- Can **solve problems** by applying their mathematics to a variety of routine and non-routine problems with increasing sophistication, including breaking down problems into a series of simpler steps and persevering in seeking solutions

Our mathematical intent is to:

- To inspire a genuine enjoyment of maths
- To develop children's conceptual understanding of mathematics through the research based approach (concrete – pictorial – abstract) to help every child master mathematical concepts
- Develop a fluent knowledge and recall of number facts and the number system
- Develop fluency in performing written and mental calculations and mathematical techniques
- Teach a broad range of skills in using and applying mathematics, including those related to solving real life problems
- Teach a wide range of mathematical vocabulary
- Develop the ability to reason, generalise and make sense of solutions
- Encourage the ability to think independently and to persevere when faced with challenges
- Provide opportunities to apply mathematical strategies and understanding to other subjects notably science and geography
- For pupils to be offered rich, sophisticated problems to develop greater depth and mastery of content
- To ensure a consistent approach to teaching across the school and at home by sharing year group expectations and informing parents and carers about methodology used in the classroom.

Implementation Strategy

At St Joseph's, mathematics is taught in line with the National Curriculum. Each year group follows the White Rose Maths long-term and medium term planning documents (see appendix 1) to ensure thorough coverage of the National Curriculum objectives.

Planning

All unit plans cover the objectives set out in the medium term plan and are saved on the Public drive alongside resources designed for each lesson. The White Rose Maths scheme breaks down the NC objectives into small steps. Teachers follow the small-step learning journey when they plan a unit. Teachers also use concrete resources to secure learning such as 10 Frames, part-whole templates, place value counters and Base Ten equipment.

Differentiation –

All lessons are designed to enable children of all abilities to access the learning and enable the most able to thrive. Maths lessons are differentiated and tailored to support the learning needs of the children. This may be in the form of adult support, task-based differentiation or scaffolding using resources. Teachers provide a challenge to extend and challenge the thinking of the most able students each lesson.

The Teaching of Calculations

The teaching of the four operations is to follow our school's calculations policy (see appendix 2).

Language

Children are encouraged to discuss mathematical concepts, to help them turn their thoughts into concrete ideas and reason well. Discourse and working with others is a vital part of Vygotsky's zone of proximal development, helping to scaffold children's learning further. To support this, children are encouraged to debate mathematical ideas within lessons.

Recording

Each lesson, is represented in books with a Learning Objective Books show clear calculation processes which then highlight potential misconceptions for the teacher or teaching assistant to 'mark in the moment' and address so that the child can embed correct knowledge. Where practical resources have been used, an adult may record in books using a photograph.

Presentation

In their Maths books, all children:

- write in pencil,
- use a ruler when underlining, drawing margins or when straight lines are needed,
- write 1 digit per square *e.g. 3 digit number uses 3 squares* and

In addition, KS2 children must:

- draw a margin on every new page; 2 whole squares from the edge of the page and
- draw a middle margin - 1 square wide where appropriate.

Assessment & Reporting

Regular assessment will be carried out through:

- 'In the moment' marking (verbal & written); instant child conferencing whilst the child is working.
- Regular marking; following the school's marking policy.
- Teachers assess the learning at the end of each lesson by highlighting the 'Learning Objective' green if the child has succeeded and pink if not. Where a child has been assessed as not achieving the objective, the lesson must be followed up with further support.
- At the beginning and end of each maths unit of work, children complete a 'cold' and 'hot' test in order to measure the progress of each teaching unit.
- NFER standardised tests are used as summative assessments each term in years 1,3,4,5. Key stage 1 and 2 SATS tests are used in year 2 and 6 to measure progress. The results of these tests are reported on the school assessment system and to parents.

Role of the Subject Leader

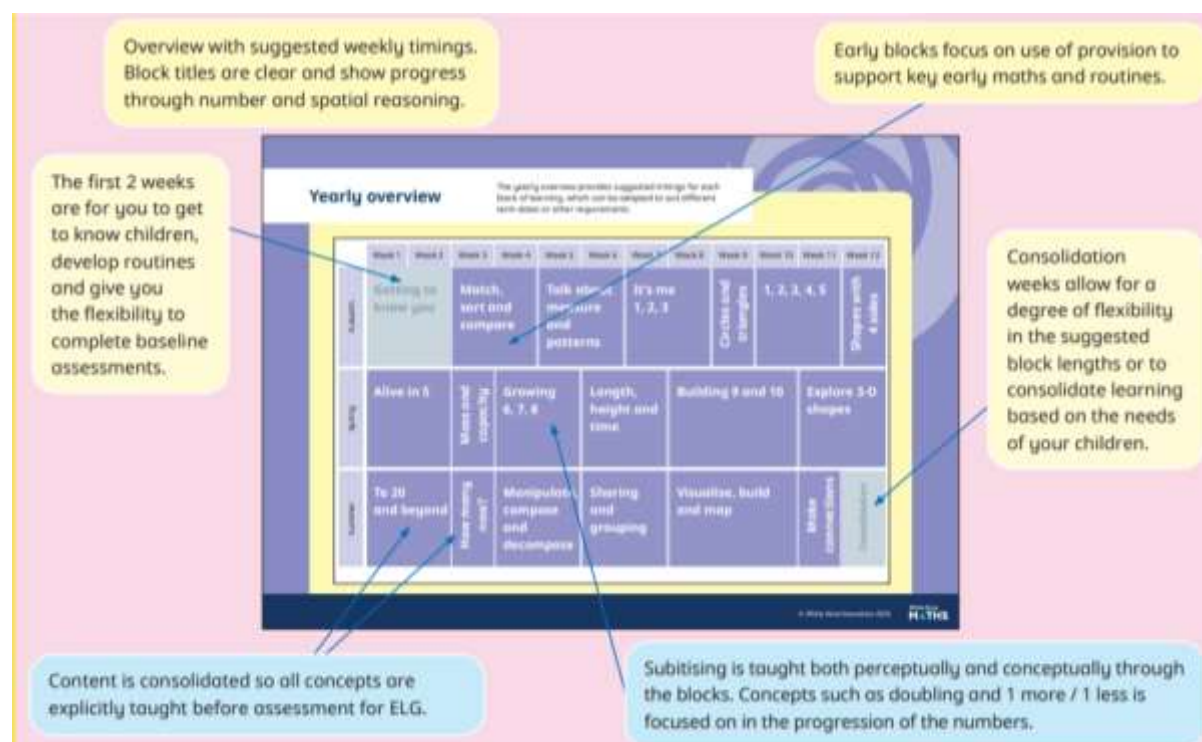
The Mathematics Subject Leader will:-

- monitor plans and sample children's books on a regular basis,
- observe classroom practice and give constructive feedback,
- organise INSET and lead staff meetings,
- provide annual budget bids,
- audit and maintain resources,
- review the Mathematics Policy and current practice in line with the NC in consultation with staff,
- analyse whole school data and report findings to staff,
- inform and work with our school governors including an annual subject leader report,
- gather pupil voice evidence and
- networking with other schools/Catholic partnership.

Each classroom will have a designated mathematics working wall. These displays are not put up in their entirety at the beginning of a unit of work, they evolve and develop as the children's work progresses. At the beginning of the unit, they often look bare but have key features of the mathematics unit of work the children are currently working on. Throughout the unit, examples of successful work or strategies to solve questions will be displayed as reminders for the children but also as a scaffold.

Appendix 1 – White Rose Yearly Overview

EYFS Overview -



Year 1 Overview –



Year 2 Overview -

	Week 1	Week 2	Week 3	Week 4	Week 5	Week 6	Week 7	Week 8	Week 9	Week 10	Week 11	Week 12
Autumn	Number Place value				Number Addition and subtraction				Geometry Shape			
Spring	Measurement Money	Number Multiplication and division						Measurement Length and height	Measurement Mass, capacity and temperature			
Summer	Number Fractions			Measurement Time			Statistics		Geometry Position and direction	Consolidation		

Year 3 Overview -

	Week 1	Week 2	Week 3	Week 4	Week 5	Week 6	Week 7	Week 8	Week 9	Week 10	Week 11	Week 12
Autumn	Number Place value			Number Addition and subtraction				Number Multiplication and division A				
Spring	Number Multiplication and division B			Measurement Length and perimeter			Number Fractions A			Measurement Mass and capacity		
Summer	Number Fractions B	Measurement Money	Measurement Time			Geometry Shape			Statistics		Consolidation	

Year 4 Overview –

	Week 1	Week 2	Week 3	Week 4	Week 5	Week 6	Week 7	Week 8	Week 9	Week 10	Week 11	Week 12
Autumn	Number Place value				Number Addition and subtraction			Measurement Area	Number Multiplication and division A			Consolidation
Spring	Number Multiplication and division B			Measurement Length and perimeter		Number Fractions			Number Decimals A			
Summer	Number Decimals B		Measurement Money		Measurement Time		Consolidation	Geometry Shape		Statistics	Geometry Position and direction	

Year 5 Overview –

	Week 1	Week 2	Week 3	Week 4	Week 5	Week 6	Week 7	Week 8	Week 9	Week 10	Week 11	Week 12
Autumn	Number Place value			Number Addition and subtraction		Number Multiplication and division A			Number Fractions A			
Spring	Number Multiplication and division B			Number Fractions B		Number Decimals and percentages			Measurement Perimeter and area		Statistics	
Summer	Geometry Shape			Geometry Position and direction		Number Decimals			Number Negative numbers	Measurement Converting units		Measurement Volume

Year 6 Overview –

	Week 1	Week 2	Week 3	Week 4	Week 5	Week 6	Week 7	Week 8	Week 9	Week 10	Week 11	Week 12	
Autumn	Number Place value		Number Addition, subtraction, multiplication and division					Number Fractions A		Number Fractions B		Measurement Converting units	
Spring	Ratio		Algebra		Number Decimals		Number Fractions, decimals and percentages		Measurement Area, perimeter and volume		Statistics		
Summer	Geometry Shape			Geometry Position and direction	Themed projects, consolidation and problem solving								

Progression in calculations

Year 1

National curriculum objectives linked to addition and subtraction

These objectives are explicitly covered through the strategies outlined in this document:

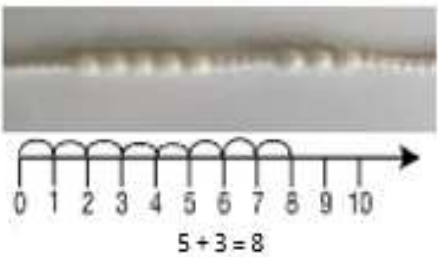
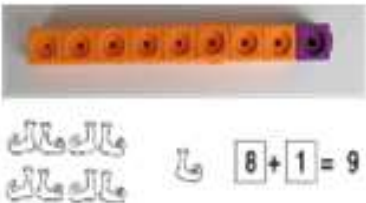
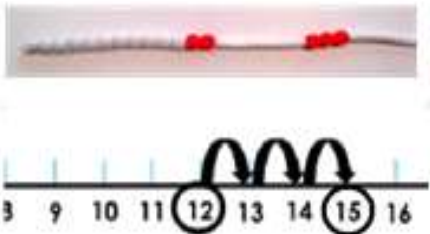
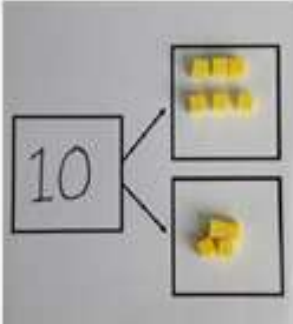
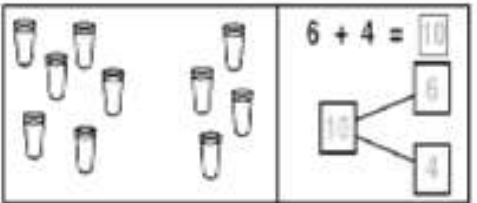
- ☐ Add and subtract one-digit and two-digit numbers to 100, including zero (N.B. Year 1 N.C. objective is to do this with numbers to 20).
- ☐ Add and subtract numbers using concrete objects, pictorial representations, and mentally, including: a two-digit number and ones, a two-digit number and tens, 2 two-digit numbers; add 3 one-digit numbers (Year 2).
- ☐ Represent and use number bonds and related subtraction facts within 20.
- ☐ Given a number, identify 1 more and 1 less.
- ☐ Show that addition of two numbers can be done in any order (commutative) but subtraction of one number from another cannot (Year 2).
- ☐ Recognise the inverse relationship between addition and subtraction and use this to solve missing number problems (Year 2).

The following objectives should be planned for lessons where new strategies are being introduced and developed:

- ☐ Read, write and interpret mathematical statements involving addition (+), subtraction (–) and equal (=) signs.
- ☐ Solve one-step problems that involve addition and subtraction, using concrete objects and pictorial representations, and missing number problems, such as $7 = \square - 9$.
- ☐ Solve problems with addition and subtraction:
 - o Using concrete objects and pictorial representations, including those involving numbers, quantities and measures
 - o Applying their increasing knowledge of mental methods

Teachers should refer to the definitions and guidance on the structures for addition and subtraction to provide a range of appropriate real-life contexts for calculations.

Y1 Addition

Strategy & guidance	CPA	
Count all <i>Joining two groups and then recounting all objects using one-to-one correspondence</i>	$3 + 4 = 7$ 	
Counting on <i>As a strategy, this should be limited to adding small quantities only (1, 2 or 3) with pupils understanding that counting on from the greater number is more efficient.</i>	$8 + 1 = 9$ 	$15 = 12 + 3$ 
Part-part-whole <i>Teach both addition and subtraction alongside each other, as pupils will use this model to identify the inverse relationship between them.</i> <i>This model begins to develop the understanding of the commutativity of addition, as pupils become aware that the parts will make the whole in any order.</i>	  $10 = 6 + 4$ $10 - 6 = 4$ $10 - 4 = 6$ $10 = 4 + 6$	

Regrouping ten ones to make ten

This is an essential skill that will support column addition later on.



$$3 + 9 =$$

$$3 + 9 = 12$$



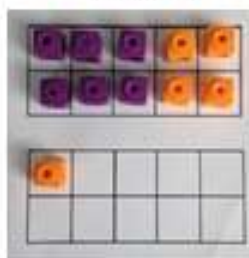
'Make ten' strategy

Pupils should be encouraged to start at the greater number and partition the smaller number to make ten.

The colours of the beads on the bead string make it clear how many more need to be added to make ten.

Also, the empty spaces on the ten frame make it clear how many more are needed to make ten.

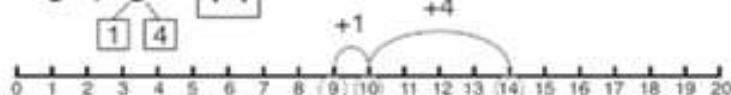
$$6 + 5 = 11$$



$$4 + 9 = 13$$



$$9 + 5 = 14$$



$$17 + 6 = 23$$

$$\begin{array}{|c|c|} \hline 3 & 3 \\ \hline \end{array}$$

Adding 1, 2, 3 more

Here the emphasis should be on the language rather than the strategy. As pupils are using the beadstring, ensure that they are explaining using language such as:

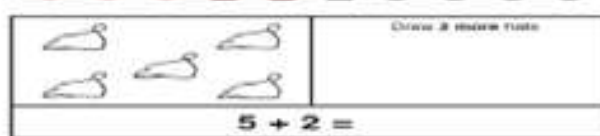
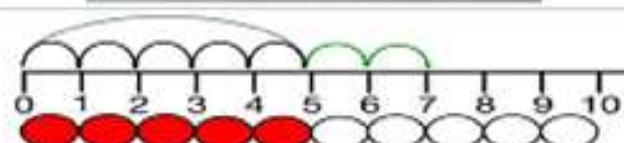
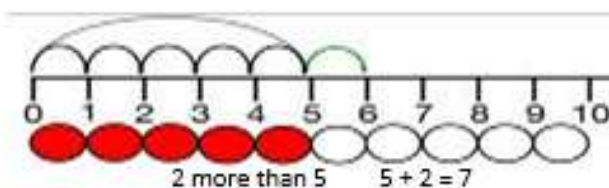
'1 more than 5 is equal to 6.'

'2 more than 5 is equal to 7.'

'8 is 3 more than 5.'

Over time, pupils should be encouraged to rely more on their number bonds knowledge than on counting strategies.

1 more than 5 $5 + 1 = 6$



Adding three single digit numbers (make ten first)

Pupils may need to try different combinations before they find the two numbers that make 10.

The first bead string shows 4, 7 and 6. The colours of the bead string show that it makes more than ten.

The second bead string shows 4, 6 and then 7.

The final bead string shows how they have now been put together to find the total.



$$\begin{array}{r} \textcircled{4} + 7 + \textcircled{6} = \boxed{10} + \boxed{7} \\ \quad \quad \quad 10 \\ \quad \quad \quad = \boxed{17} \end{array}$$

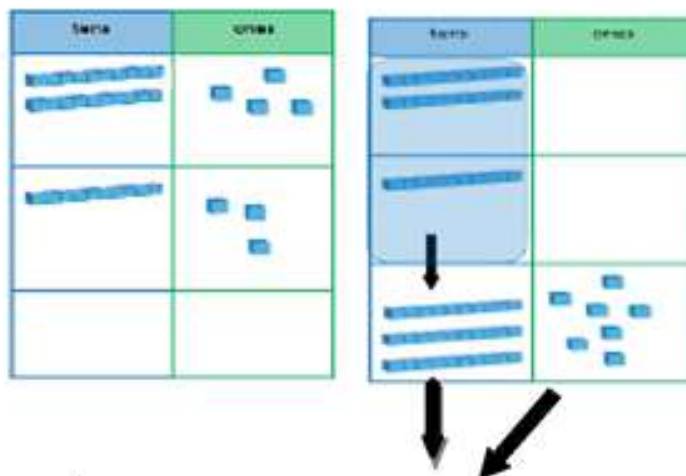
Partitioning to add (no regrouping)

Place value grids and Dienes blocks could be used as shown in the diagram before moving onto pictorial representations. Dienes blocks should always be available, as the main focus in Year 1 is the concept of place value rather than mastering the procedure.

When not regrouping,

partitioning is a mental strategy and does not need formal recording in columns. This representation prepares them for using column addition with formal recording.

$$24 + 13 = 37$$

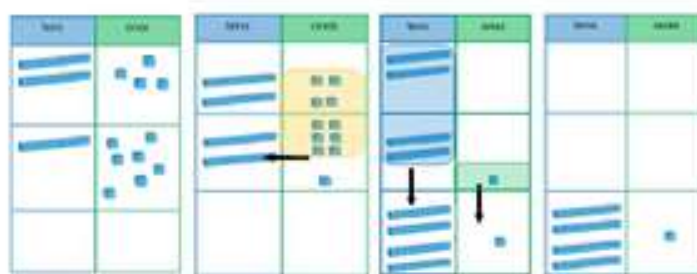


$$30 + 7 = 37$$

Introducing column method for addition, regrouping only

Dienes blocks and place value grids should be used as shown in the diagrams. Even when working pictorially, pupils should have access to Dienes blocks.

$$24 + 17$$



Tens Ones

$$\begin{array}{r} 24 \\ + 17 \\ \hline 41 \\ \hline 1 \end{array}$$

First add the ones.
Re-group 10 ones to 1 ten.
Next add the tens.



Adding multiples of ten

Using the vocabulary of 1 ten, 2 tens, 3 tens etc. **alongside** 10, 20, 30 is important, as pupils need to understand that it is a **ten** and not a one that is being added and they need to understand that a '2' digit in the tens column has a value of twenty.

It also emphasises the link to known number facts. E.g. '2 + 3 is equal to 5. So 2 tens + 3 tens is equal to 5 tens.

$$50 = 30 + 20$$



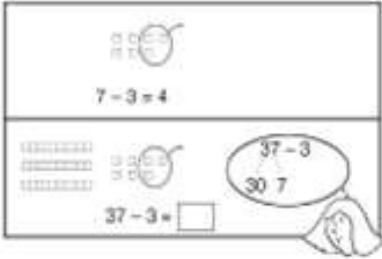
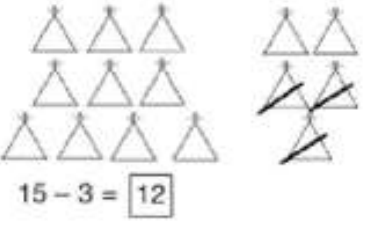
$$3 \text{ tens} + 5 \text{ tens} = \underline{\quad\quad} \text{ tens}$$

$$30 + 50 = \underline{\quad\quad}$$



$$36 + 40 = \boxed{\quad\quad}$$

Y1 Subtraction

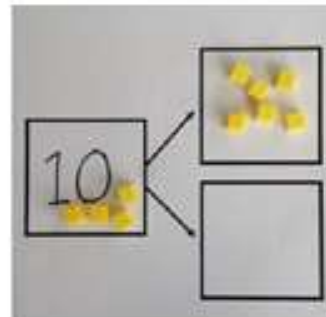
Strategy & guidance	CPA
Taking away from the ones <i>When this is first introduced, the concrete representation should be based upon the diagram. Real objects should be placed on top of the images as one-to-one correspondence so that pupils can take them away, progressing to representing the group of ten with a tens rod and ones with ones cubes.</i>	 $7 - 3 = 4$ $37 - 3 = 30$ $37 - 3 = \square$  $15 - 3 = 12$  $60 - 20 = 40$  $28 - 4 =$
Counting back <i>Subtracting 1, 2, or 3 by counting back</i> <i>Pupils should be encouraged to rely on number bonds knowledge as time goes on, rather than using counting back as their main strategy.</i>	 $4 = 6 - 2$  $16 - 2 = 14$  12 13 14 15 16 17 18

Part-part-whole

Teach both addition and subtraction alongside each other, as the pupils will use this model to identify the link between them. Pupils start with ten cubes placed on the whole.

They then remove what is being taken away from the whole and place it on one of the parts. The remaining cubes are the other part and also the answer. These can be moved into the second part space.

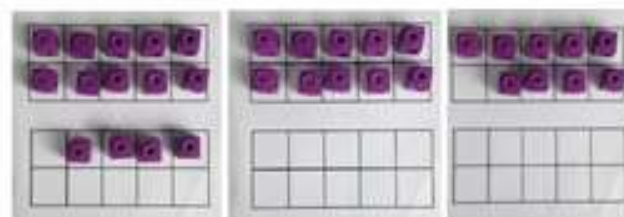
$$10 - 6 = 4$$

**Make ten strategy**

To subtract a 1-digit number from a 2-digit number.

Pupils identify how many need to be taken away to make ten first, partitioning the number being subtracted. Then they take away the rest to reach the answer.

$$14 - 5 = 9$$

**Regroup a ten into 10 ones**

After the initial introduction, the Dienes blocks should be placed on a place value chart to support place value understanding. This will support pupils when they later use the column method.



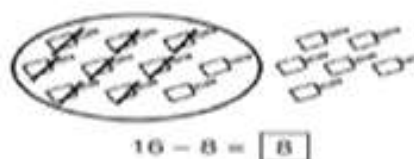
$$20 - 4 =$$

Taking away from the tens

Pupils should identify that they can also take away from the tens and get the same answer.

This reinforces their knowledge of number bonds to 10 and develops their application of number bonds for mental strategies.

$$9 = 15 - 6$$



Partitioning to subtract without regrouping

Dienes blocks on a place value chart (developing into using images on the chart) could be used, as when adding 2-digit numbers, reinforcing the main concept of place value for Year 1.

$$34 - 13 = 21$$



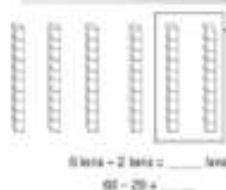
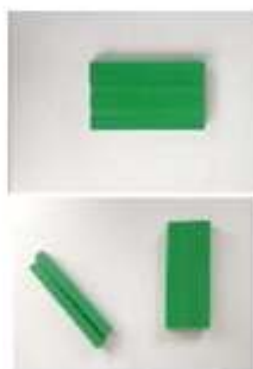
$$34 - 13 = 21$$

When not regrouping, partitioning is a mental strategy and does not need formal recording in columns. This representation prepares them for using column subtraction with formal recording.

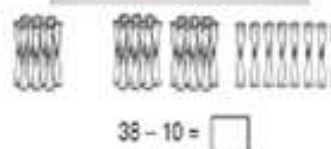
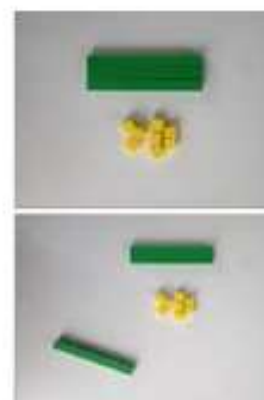
Subtracting multiples of ten

Using the vocabulary of 1 ten, 2 tens, 3 tens etc. alongside 10, 20, 30 is important as pupils need to understand that it is a **ten** not a one that is being taken away.

$$40 = 60 - 20$$



$$38 - 10 = 28$$



Column method with regrouping

This example shows how pupils should work practically when being introduced to this method.

There is no formal recording in columns in Year 1 but this practical work will prepare pupils for formal methods in Year 2.

See additional guidance on unit pages to support with this method.

$$34 - 17 = 17$$

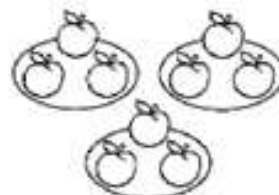


Y1 Multiplication

Strategy & guidance	CPA
Skip counting in multiples of 2, 5, 10 from zero <i>The representation for the amount of groups supports pupils' understanding of the written equation. So two groups of 2 are 2, 4. Or five groups of 2 are 2, 4, 6, 8, 10.</i> <i>Count the groups as pupils are skip counting.</i> <i>Number lines can be used in the same way as the bead string.</i> <i>Pupils can use their fingers as they are skip counting.</i>	$4 \times 5 = 20$ $2 \times 4 = 8$
Making equal groups and counting the total <i>How this would be represented as an equation will vary. This could be 2×4 or 4×2. The importance should be placed on the vocabulary used alongside the equation. So this picture could represent 2 groups of 4 or 4 twice.</i>	$\square \times \square = 8$ Draw  to show $2 \times 3 = 6$

Solve multiplications using repeated addition

$$3 \times 3 = 3 + 3 + 3$$



How many apples are there altogether?

$$3 + 3 + 3 = 9$$

Y1 Division

Strategy & guidance

CPA

Sharing objects into groups

$$10 \div 2 = 5$$

Pupils should become familiar with division equations through working practically.

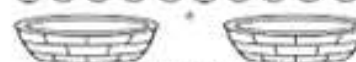
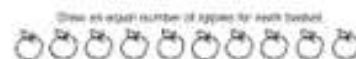
The division symbol is not formally taught at this stage.



There are 10 sweets. Ring groups of 2.



There are _____ groups of 2.



There are five apples in each basket.

Progression in calculations

Year 2

National Curriculum objectives linked to addition and subtraction

These objectives are explicitly covered through the strategies outlined in this document:

- ☐ Add and subtract numbers using concrete objects, pictorial representations, and mentally, including: a two-digit number and ones; a two-digit number and tens; 2 two-digit numbers; adding three one-digit numbers.
- ☐ Add and subtract numbers mentally, including: a three-digit number and ones; a three-digit number and tens; a three-digit number and hundreds (Year 3).
- ☐ Recall and use addition and subtraction facts to 20 fluently, and derive and use related facts up to 100.
- ☐ Find 10 or 100 more or less than a given number (Year 3).
- ☐ Show that addition of two numbers can be done in any order (commutative) but subtraction of one number from another cannot.
- ☐ Recognise and use the inverse relationship between addition and subtraction and use this to check calculations and solve missing number problems.
- ☐ Add and subtract numbers with up to three digits, using formal written methods of columnar addition and subtraction (Year 3).

The following objectives should be planned for lessons where new strategies are being introduced and developed:

- ☐ Solve problems with addition and subtraction: using concrete objects and pictorial representations, including those involving numbers, quantities and measures; apply increasing knowledge of mental and written methods.
- ☐ Solve problems, including missing number problems, using number facts, place value and more complex addition and subtraction. (Year 3)

Teachers should refer to the definitions and guidance on the structures for addition and subtraction to provide a range of appropriate real-life contexts for calculations.

Y2 Addition

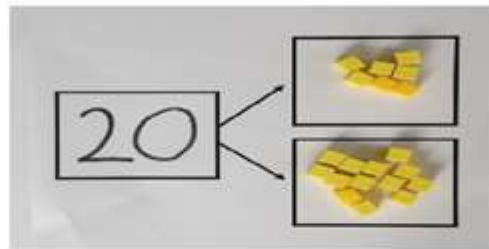
Strategy & guidance

Part-part-whole

Pupils explore the different ways of making 20. They can do this with all numbers using the same representations.

This model develops knowledge of the inverse relationship between addition and subtraction and is used to find the answer to missing number problems.

CPA

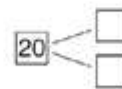
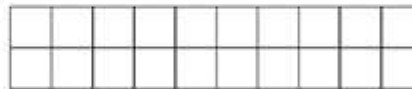


$$20 = 17 + 3$$

$$20 = 3 + 17$$

$$20 - 3 = 17$$

$$20 - 17 = 3$$

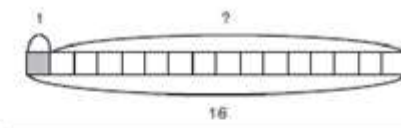


$$\square + \square = 20 \quad 20 - \square = \square$$

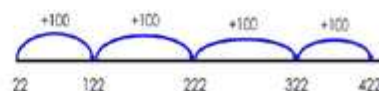
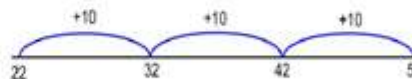
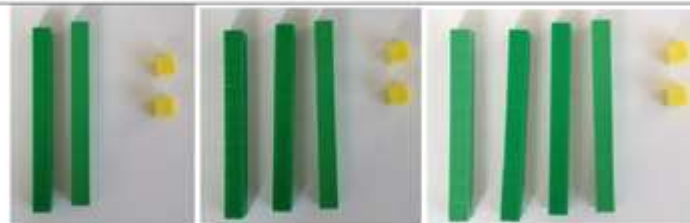
$$\square + \square = 20 \quad 20 - \square = \square$$

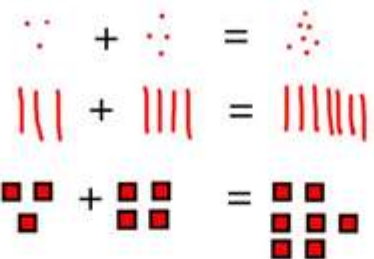
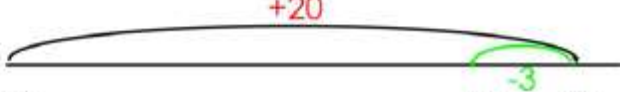
$$\square + 1 = 16 \quad 16 - 1 = \square$$

$$1 + \square = 16 \quad 16 - \square = 1$$



Counting on in tens and hundreds



Strategy & guidance	CPA
Using known facts to create derived facts <i>Dienes blocks should be used alongside pictorial and abstract representations when introducing this strategy.</i>	 $3 + 4 = 7$ leads to $30 + 40 = 70$ leads to $300 + 400 = 700$
Partitioning one number, then adding tens and ones <i>Pupils can choose themselves which of the numbers they wish to partition. Pupils will begin to see when this method is more efficient than adding tens and taking away the extra ones, as shown.</i>	  22 32 39 $22 + 17 = 39$
Round and adjust (sometimes known as a compensating strategy) <i>Pupils will develop a sense of efficiency with this method, beginning to see when rounding and adjusting is more efficient than adding tens and then ones.</i>	  22 39 42 $22 + 17 = 39$

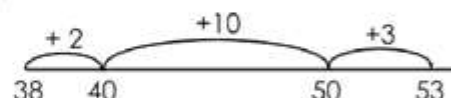
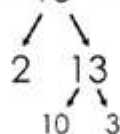
Strategy & guidance**CPA****Make ten strategy**

How pupils choose to apply this strategy is up to them; however, the focus should always be on efficiency.

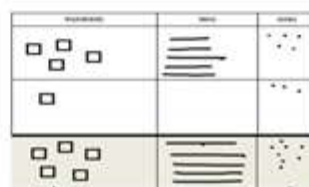
It relies on an understanding that numbers can be partitioned in different ways in order to easily make a multiple of ten.



$$38 + 15 =$$

**Partitioning to add without regrouping**

As in Year 1, this is a mental strategy rather than a formal written method. Pupils use the Dienes blocks (and later, images) to represent 3-digit numbers but do not record a formal written method if there is no regrouping.



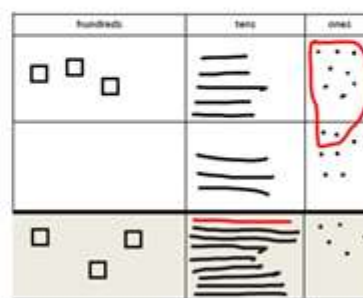
$$455 + 103 = 558$$

Column method with regrouping

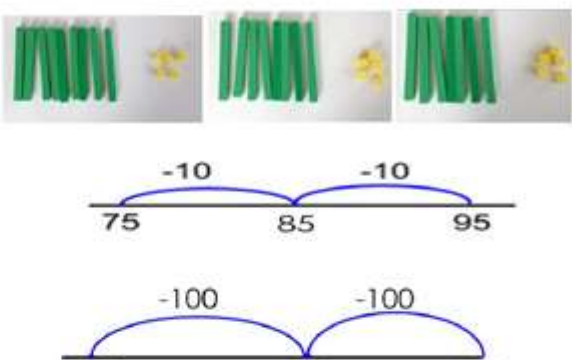
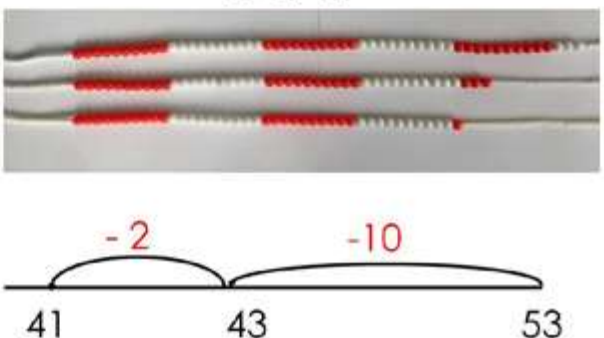
Dienes blocks should be used alongside the pictorial representations; they can be placed on the place value grid before pupils make pictorial representations.

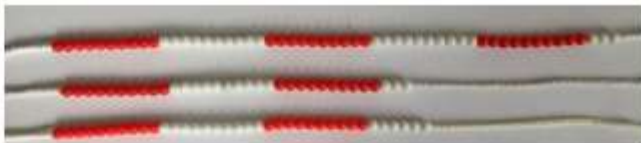
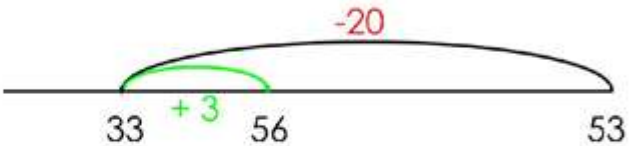
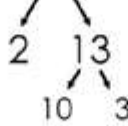
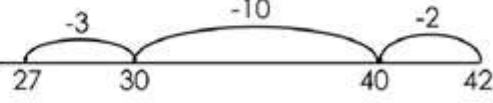
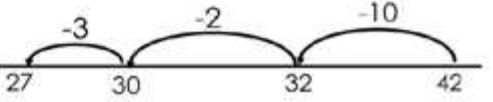
As in Year 1, the focus for the column method is to develop a strong understanding of place value.

hundreds	tens	ones
3	5	8
3	3	7
3	9	5



Y2 Subtraction

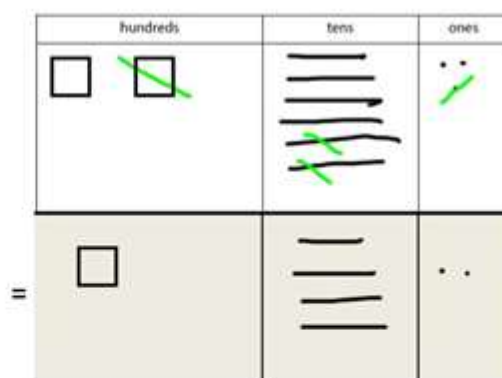
Strategy & guidance	CPA
Counting back in multiples of ten and one hundred	
Using known number facts to create derived facts <i>Dienes blocks should be used alongside pictorial and abstract representations when introducing this strategy, encouraging pupils to apply their knowledge of number bonds to add multiples of ten and 100.</i>	 $8 - 4 = 4$ <i>leads to</i> $80 - 40 = 40$ <i>leads to</i> $800 - 400 = 400$
Subtracting tens and ones <i>Pupils must be taught to partition the second number for this strategy as partitioning both numbers can lead to errors if regrouping is required.</i>	$53 - 12 = 41$ 

Strategy & guidance	CPA
Round and adjust (sometimes known as a compensating strategy) <i>Pupils must be taught to round the number that is being subtracted.</i> <i>Pupils will develop a sense of efficiency with this method, beginning to identify when this method is more efficient than subtracting tens and then ones.</i>	  $53 - 17 = 36$
Make ten <i>How pupils choose to apply this strategy is up to them. The focus should always be on efficiency.</i> <i>It relies on an understanding that numbers can be partitioned in different ways in order to subtract to a multiple of ten.</i> <i>Pupils should develop an understanding that the parts can be added in any order.</i>	 $42 - 15 =$   

Strategy & guidance**Partitioning to subtract without regrouping**

As in Year 1, the focus is to develop a strong understanding of place value and pupils should always be using concrete manipulatives alongside the pictorial.

Formal recording in columns is unnecessary for this mental strategy. It prepares them to subtract with 3-digits when regrouping is required.

CPA

$$263 - 121 = 142$$

Column method with regrouping

The focus for the column

strong understanding of place value and concrete manipulatives should be used alongside.

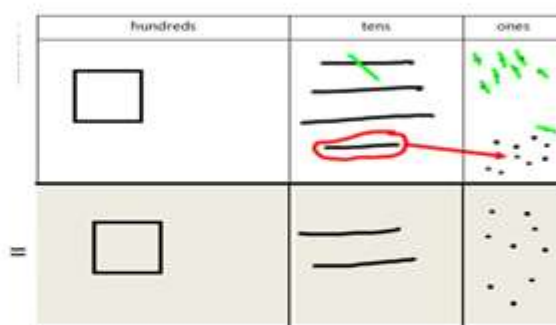
Pupils are introduced to calculations that require two instances of regrouping (initially from tens to one and then from hundreds to tens). E.g. 232

- 157 and are given plenty of practice using concrete manipulatives and images alongside their formal written methods, ensuring that important steps are not missed in the recording.

Caution should be exercised when introducing calculations requiring 'regrouping to regroup' (e.g. 204 - 137) ensuring ample teacher modelling using concrete manipulatives and images.

hundreds tens ones

$$\begin{array}{r} 1^{3}4^{1}7 \\ - 18 \\ \hline 129 \end{array}$$



National Curriculum objectives linked to multiplication and division

These objectives are explicitly covered through the strategies outlined in this document:

- ☐ Recall and use multiplication and division facts for the 2, 5 and 10 multiplication tables, including recognising odd and even numbers.
- ☐ Recall and use multiplication and division facts for the 3 and 4 multiplication tables (Year 3).
- ☐ Show that multiplication of two numbers can be done in any order (commutative) but division of one number by another cannot.

The following objectives should be planned for lessons where new strategies are being introduced and developed:

- ☐ Calculate mathematical statements for multiplication and division within the multiplication tables and write them using the multiplication ($\times$), division ($\div$) and equal ($=$) signs.
- ☐ Solve problems involving multiplication and division, using materials, arrays, repeated addition, mental methods and multiplication and division facts, including problems in context.

Teachers should refer to definitions and guidance on the structures for multiplication and division to provide a range of appropriate real-life contexts for calculations.

Y2

Strategy & guidance

Skip counting in multiples of 2, 3, 4, 5, 10 from zero

Pupils can use their fingers as they are skip counting, to develop an understanding of 'groups of'.

Dot arrays can be used to create a visual representation for the different multiplication facts. Bead strings, groups of cubes (or ~~unifix~~ / multilink towers) provide useful concrete representations.

Dot grid paper (multiplication table of 5)

	1	2	3	4	5
0					
1	●	●	●	●	●
2	●	●	●	●	●
3	●	●	●	●	●
4	●	●	●	●	●
5	●	●	●	●	●
6	●	●	●	●	●
7	●	●	●	●	●
8	●	●	●	●	●
9	●	●	●	●	●
10	●	●	●	●	●
11	●	●	●	●	●
12	●	●	●	●	●

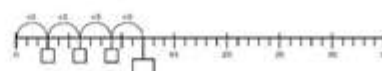


Multiplication as repeated addition

Pupils apply skip counting to help find the totals of repeated additions.



$$5 + 5 + 5 + 5 + 5 + 5 + 5 + 5 = \square$$



$$4 \times 3 = \square$$

Strategy & guidance**Arrays to represent multiplication equations**

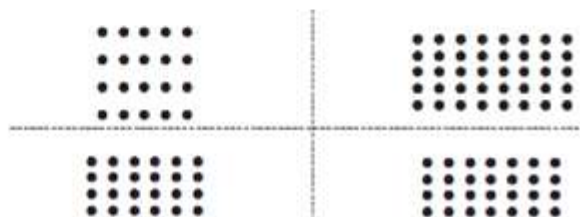
Concrete manipulatives and images of familiar objects begin to be organised into arrays and, later, are shown alongside dot arrays. It is important to discuss with pupils how arrays can be useful.

Pupils begin to understand multiplication in a more abstract fashion, applying their skip counting skills to identify the multiples of the 2x, 5x and 10x tables.

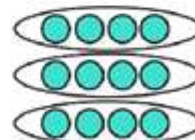
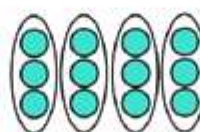
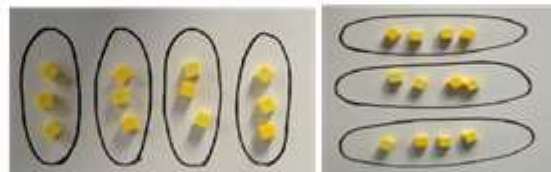
The relationship between multiplication and division also begins to be demonstrated.

Multiplication is commutative

Pupils should understand that an array and, later, bar models can represent different equations and that, as multiplication is commutative, the order of the multiplication does not affect the answer.

CPA

$$3 \times 5 = \square$$
$$5 \times 3 = \square$$



$$12 = 3 \times 4$$

$$12 = 4 \times 3$$

Strategy & guidance

CPA

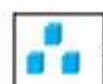
Use of part-part-whole model to establish the inverse relationship between multiplication and division

This link should be made explicit from early on, using the language of the part-part-whole model, so that pupils develop an early understanding of the relationship between multiplication and division. Bar models (with Cuisenaire rods) should be used to identify the whole, the value of the parts and the number of parts.

It is important to highlight that with multiplication, the parts are of equal value as this is different to how this model is used for addition and subtraction.

There are three equal parts. Each part has a value of three. What is the whole?

$$3 \times 3 = \square$$

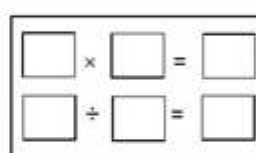
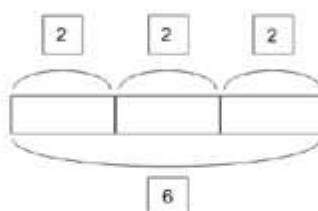


$$9 \div 3 = \square$$



What multiplication and division equations can you write for each bar model?

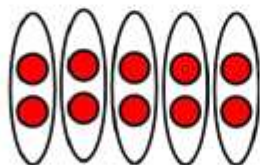
Prove that the equations are correct using a bead string.



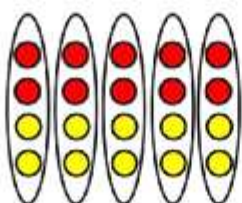
Doubling to derive new multiplication facts

Pupils learn that known facts from easier multiplication tables can be used to derive facts from related times tables using doubling as a 20 strategy.

At this stage they double the 2x table facts to derive the 4x table facts.

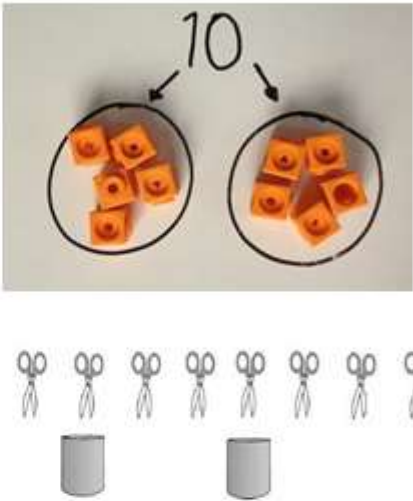
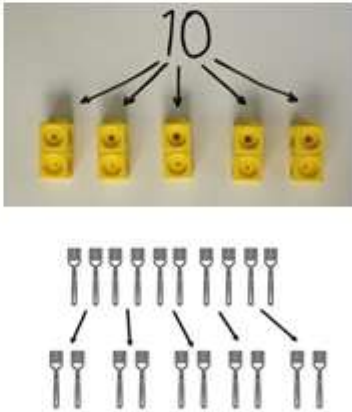


$$5 \times 2 = 10$$



$$5 \times 4 =$$

Y2 Division

Strategy & guidance	CPA
Division as sharing <i>Here, division is shown as sharing.</i> <i>If we have ten pairs of scissors and we share them between two pots, there will be 5 pairs of scissors in each pot.</i>	$10 \div 2 = 5$  The diagram illustrates division as sharing. At the top, the number '10' is written with two arrows pointing down to two separate circles. Each circle contains five orange blocks, representing a pair of scissors. Below the circles, there are ten pairs of scissors icons arranged in a row. Below the scissors, there are two grey cylindrical pots, one under each group of scissors, representing the two groups into which the scissors are being shared.
Division as grouping <i>Here, division is shown as grouping.</i> <i>If we have ten forks and we put them into groups of two, there are 5 groups.</i>	$10 \div 2 = 5$  The diagram illustrates division as grouping. At the top, the number '10' is written with five arrows pointing down to five separate groups of two yellow blocks, representing a pair of forks. Below the blocks, there are ten forks icons arranged in a row. Below the forks, there are five groups of two forks each, with arrows pointing from the top row of forks to these groups, representing the five groups of two.

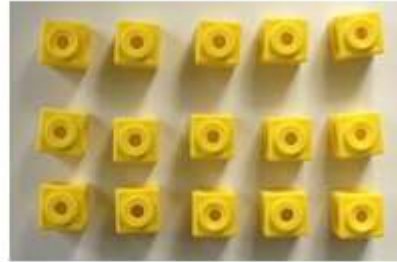
CPA

Use of part-part-whole model to represent division equations and to emphasise the relationship between division and multiplication

Pupils use arrays of concrete manipulatives and images of familiar objects to solve division equations.

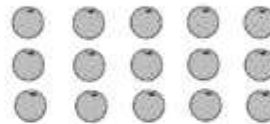
They begin to use dot arrays to develop a more abstract concept of division.

It is important to highlight that with multiplication and division, the parts are of equal value as this is different to how this model is used for addition and subtraction.



$$15 \div 5 = \boxed{3}$$

$$15 \div 3 = \boxed{5}$$



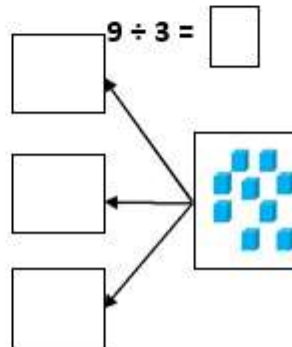
Write the division equations that the array represents.



$$20 \div 4 = \boxed{}$$

$$20 \div 5 = \boxed{}$$

The whole is nine. There are three equal parts. What is the value of each part?



Progression in calculations

Year 3

National Curriculum objectives linked to addition and subtraction

These objectives are explicitly covered through the strategies outlined in this document:

- ☐ add and subtract numbers mentally, including:
 - o a three-digit number and ones
 - o a three-digit number and tens
 - o a three-digit number and hundreds
- ☐ add and subtract numbers with up to four digits, using formal written methods of columnar addition and subtraction (four digits is Year 4)
- ☐ find 10 or 100 more or less than a given number
- ☐ find 1 000 more or less than a given number (Year 4)
- ☐ estimate the answer to a calculation and use inverse operations to check answers

The following objectives should be planned for lessons where new strategies are being introduced and developed:

- ☐ solve problems, including missing number problems, using number facts, place value, and more complex addition and subtraction

Teachers should refer to definitions and guidance on the structures for addition and subtraction to provide a range of appropriate real-life contexts for calculations.

Y3 Addition & Subtraction

Strategy & guidance	CPA														
Add and subtract numbers mentally, including: • a three-digit number and ones; • a three-digit number and tens; • a three-digit number and hundreds <i>Pupils learn that this is an appropriate strategy when they are able to use known and derived number facts or other mental strategies to complete mental calculations with accuracy.</i> <i>To begin with, some pupils will prefer to use this strategy only when there is no need to regroup, using number facts within 10 and derivations. More confident pupils might choose from a range of mental strategies that avoid written algorithms, including (but not exhaustively):</i> • known number facts within 20, • derived number facts, • 'Make ten', • round and adjust <i>See Year 2 guidance for exemplification of these – the use of concrete manipulatives other than Dienes blocks is important in reinforcing the use of these strategies.</i> <i>It is important that pupils are given plenty of (scaffolded) practice at choosing their own strategies to complete calculations efficiently and accurately. Explicit links need to be made between familiar number facts and the calculations that they can be useful for and pupils need to be encouraged to aim for efficiency.</i>	It is important to model the mental strategy using concrete manipulatives in the first instance and pupils should be able to exemplify their own strategies using manipulatives if required. However, pupils should be encouraged to use known facts to derive answers, rather than relying on counting manipulatives or images. <u>No regrouping</u> <table> <tr> <td>$345 + 30$</td><td>$274 - 50$</td></tr> <tr> <td>$1128 + 300$</td><td>$1312 - 300$</td></tr> <tr> <td>$326 + 342$</td><td>$856 - 724$</td></tr> </table>  I know $4 + 3 = 7$, so 4 tens plus 3 tens is equal to 7 tens. $345 + 30 = 375$. <u>With some regrouping</u> <table> <tr> <td>$416 + 25$</td><td>$232 - 5$</td></tr> <tr> <td>$383 + 130$</td><td>$455 - 216$</td></tr> <tr> <td>$611 + 194$</td><td>$130 - 40$</td></tr> <tr> <td>$1482 + 900$</td><td>$2382 - 500$</td></tr> </table>	$345 + 30$	$274 - 50$	$1128 + 300$	$1312 - 300$	$326 + 342$	$856 - 724$	$416 + 25$	$232 - 5$	$383 + 130$	$455 - 216$	$611 + 194$	$130 - 40$	$1482 + 900$	$2382 - 500$
$345 + 30$	$274 - 50$														
$1128 + 300$	$1312 - 300$														
$326 + 342$	$856 - 724$														
$416 + 25$	$232 - 5$														
$383 + 130$	$455 - 216$														
$611 + 194$	$130 - 40$														
$1482 + 900$	$2382 - 500$														

Strategy & guidance**Written column method for calculations that require regrouping with up to 4-digits**

Dienes blocks should be used alongside the pictorial representations during direct teaching and can be used by pupils both for support and challenge. Place value counters can also be introduced at this stage.

This work revises and reinforces ideas from Key

Stage 1, including the focus on place value – see Year 2 exemplification.

Direct teaching of the columnar method should require at least one element of regrouping, so that pupils are clear about when it is most useful to use it. Asking them ‘Can you think of a more efficient method?’ will challenge them to apply their number sense / number facts to use efficient mental methods where possible.

As in Year 2, pupils should be given plenty of practice with calculations that require multiple separate instances of regrouping. In Year 3 they become more familiar with calculations that require ‘regrouping to regroup’. Understanding must be secured through the considered use of manipulatives and images, combined with careful use of language.

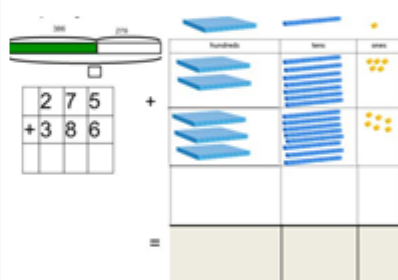
Pupils should be challenged as to whether this is the most efficient method, considering whether mental methods (such as counting on, using known number

facts, round and adjust etc.) may be likelier to produce an accurate solution.

Pupils requiring support might develop their confidence in the written method using numbers that require no regrouping.

CPA

As for the mental strategies, pupils should be exposed to concrete manipulatives modelling the written calculations and should be able to represent their written work pictorially or with concrete manipulatives when required. Again, they should be encouraged to calculate with known and derived facts and should not rely on counting images or manipulatives.



5 + 6 = 11 so I will have 11 ones which I regroup for 1 ten and 1 one.

Regrouping (including multiple separate instances)

$$672 +$$

$$136$$

$$734 - 82$$

$$468 + 67$$

$$831 - 76$$

$$275 +$$

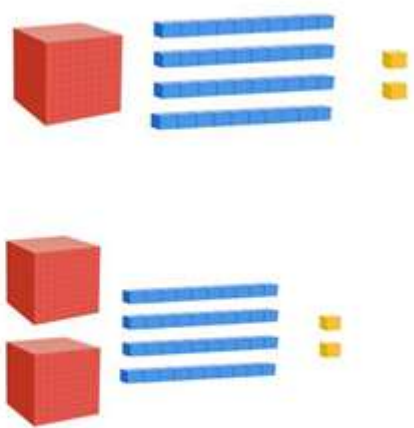
$$386$$

$$435 - 188$$

‘Regrouping to regroup’

$$204 - 137$$

$$1035 - 851$$

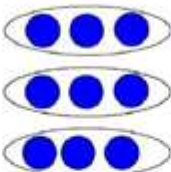
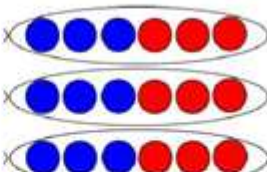
Strategy & guidance	CPA
Find 10, 100 and 1000 more or less than a given number <i>As pupils become familiar with numbers up to 1000, place value should be emphasised and comparisons drawn between adding tens, hundreds (revision - see Year 2 guidance) and thousands, including use of concrete manipulatives and appropriate images.</i> <i>After initial teaching, this should be incorporated into transition activities and practised regularly.</i>	$1042 + 1000 = 2042$  The diagram illustrates the addition of 1000 to 1042 using base ten blocks. In the top row, representing 1042, there is one red cube (1000), four blue rods (40), and two yellow units (2). In the bottom row, representing the sum 2042, there are two red cubes (2000), four blue rods (40), and two yellow units (2). This visualizes the process of adding 1000 to a number, which increases the thousands place by one.

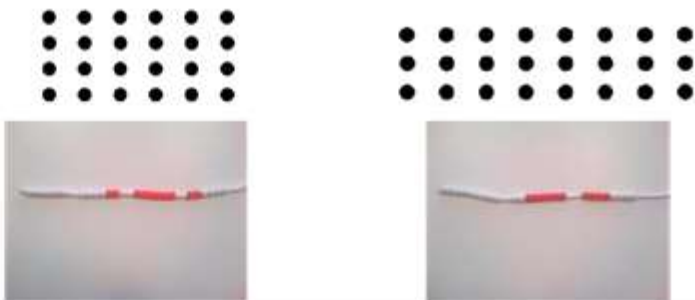
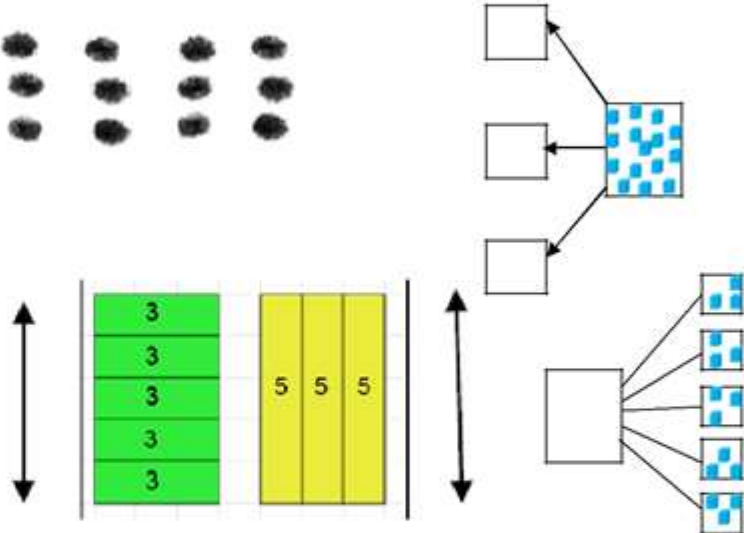
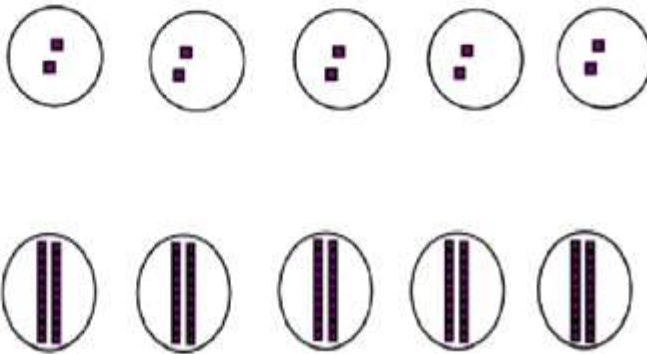
National Curriculum objectives linked to multiplication and division

These objectives are explicitly covered through the strategies outlined in this document:

- ☐ count from 0 in multiples of 4, 8, 50 and 100
- ☐ recall and use multiplication and division facts for the 3, 4, 6, and 8 multiplication tables
- ☐ write and calculate mathematical statements for multiplication and division using the multiplication tables that they know, including for two-digit numbers times one-digit numbers, using mental methods
- ☐ solve problems, including missing number problems, involving multiplication and division, including positive integer scaling problems and correspondence problems in which n objects are connected to m objects

Teachers should refer to definitions and guidance on the structures for multiplication and division to provide a range of appropriate real-life contexts for calculations.

Strategy & guidance	CPA	
Doubling to derive new multiplication facts <i>Pupils continue to make use of the idea that facts from easier times tables can be used to derive facts from related times tables using doubling as a strategy.</i> <i>This builds on the doubling strategy from Year 2.</i>	$3 \times 3 = 9$ 	$3 \times 6 = \text{double } 9 = 18$ 

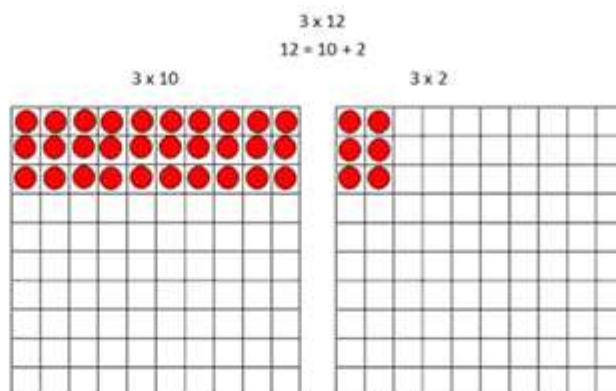
Strategy & guidance	CPA
Skip counting in multiples of 2, 3, 4, 5, 6, 8 and 10 <i>Rehearsal of previously learnt tables as well as new content for Year 3 should be incorporated into transition activities and practised regularly.</i>	
Use of part-part-whole model with arrays and bar models to establish commutativity and inverse relationship between multiplication and division <i>In these contexts, pupils are able to identify all the equations in a fact family.</i>	
Ten times greater <i>Pupils's work on this must be firmly based on concrete representations – the language of ten times greater must be well modelled and understood to prevent the numerical misconception of 'adding a zero'.</i>	

Strategy & guidance	CPA
Multiplying by 10, 100 and 1000 <i>Building on the ten times greater work, pupils use appropriate Dienes blocks and place value counters to multiply 2, 3, 4, 5 and 10 by 10, 100 and 1000.</i>	$5 \times 1 = 5$  $5 \times 10 = 50$  $3 \times 1 = 3$  $3 \times 100 = 300$ 
Using known facts for multiplying by multiples of 10, 100 and 1000 <i>Pupils' growing understanding of place value, allows them to make use of known facts to derive multiplications using powers of 10.</i> <i>It is important to use tables with which they are already familiar (i.e. not 7 or 9 tables in Year 3)</i>	$5 = 1 \times 5$  $50 = 10 \times 5$  $500 = 100 \times 5$  $5000 = 1000 \times 5$  $3 \times 2 = 6$  $30 \times 2 = 60$  $300 \times 2 = 600$  $3000 \times 2 = 6000$     

Strategy & guidance**Multiplication of 2-digit numbers with partitioning (no regrouping)**

Children should always consider whether partitioning is the best strategy – if it is possible to use strategies such as doubling (some may use doubling twice for $\times 4$), they need to choose the most efficient strategy.

Children may wish to make jottings, including a full grid as exemplified here – but grid method is not a formal method and its only purpose is to record mental calculations. This supports the development of the necessary mental calculating skills but does not hinder the introduction of formal written methods in Year 4. Concrete manipulatives are essential to develop understanding.

CPA

Now add the total number of tens and ones

$\times$	10	2
3		

$\times$	10	2
3	30	6

$$3 \times 12 = 36$$

Strategy & guidance**Multiplication of 2-digit numbers with partitioning (regrouping)**

Using concrete manipulatives and later moving to using images that represent them, supports pupils' early understanding, leading towards formal written methods in Year 4.

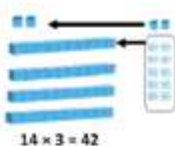
Once again, this is a mental strategy, which they may choose to support with informal jottings, including a full grid, as exemplified here.

Pupils must be encouraged to make use of their known multiplication facts and their knowledge of place value to calculate, rather than counting manipulatives.

CPA

×	10	4
3		
	30	12

×	10	4
3	30	12



×	40	5
3		



Y3 Division

Strategy & Guidance	CPA															
Dividing multiples of 10, 100 and 1000 by 10, 100 and 1000 using scaling down <i>Pupils use the strategy of 'scaling down', representing numbers with concrete manipulatives and making the value ten times smaller.</i>	<table><thead><tr><th>hundreds</th><th>tens</th><th>ones</th></tr></thead><tbody><tr><td></td><td></td><td></td></tr><tr><td></td><td> 3</td><td> 3</td></tr><tr><td></td><td></td><td>0</td></tr><tr><td></td><td></td><td> 3</td></tr></tbody></table> $3 \times 10 = 30$ $30 \div 10 = 3$	hundreds	tens	ones					 3	 3			0			 3
hundreds	tens	ones														
																
	 3	 3														
		0														
		 3														
Dividing multiples of 10, 100 and 1000 by 10, 100 and 1000 using grouping <i>Pupils divide by 10, 100 and 1000 by making groups of the divisor.</i>	$500 \div 100 = \square$ My whole is 500 and the value of the equal parts is 100. How many parts are there? <table><thead><tr><th>Thousands</th><th>hundreds</th><th>tens</th><th>ones</th></tr></thead><tbody><tr><td></td><td> 5</td><td>0</td><td>0</td></tr></tbody></table>	Thousands	hundreds	tens	ones		 5	0	0							
Thousands	hundreds	tens	ones													
	 5	0	0													

Progression in calculations

Year 4

National curriculum objectives linked to addition and subtraction

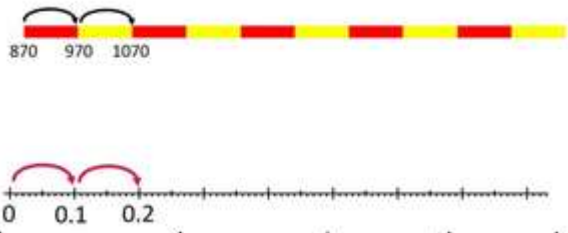
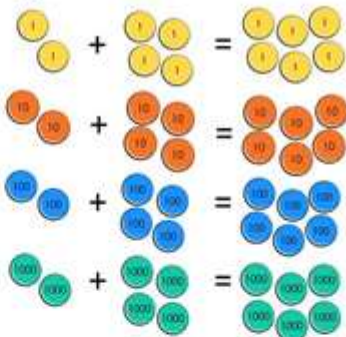
These objectives are explicitly covered through the strategies outlined in this document:

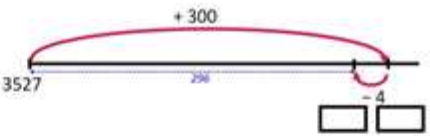
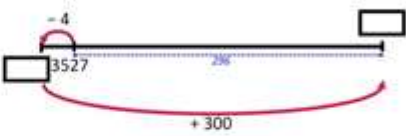
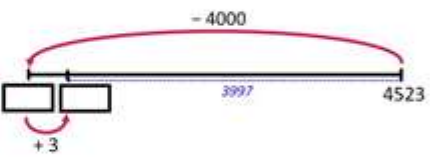
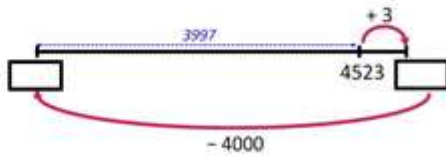
- ☐ add and subtract numbers with up to four digits, using the formal written methods of columnar addition and subtraction where appropriate
- ☐ find 1 000 more or less than a given number
- ☐ estimate and use inverse operations to check answers to a calculation

N.B. There is no explicit reference to mental calculation strategies in the programmes of study for Year 4 in the national curriculum. However, with an overall aim for fluency, appropriate mental strategies should always be considered before resorting to formal written procedures, with the emphasis on pupils making their own choices from an increasingly sophisticated range of strategies.

The following objectives should be planned for lessons where new strategies are being introduced and developed:

- ☐ solve addition and subtraction two-step problems in contexts, deciding which operations and methods to use and why
- ☐ solve simple measure and money problems involving fractions and decimals to two decimal places

Strategies & Guidance	CPA
Count forwards and backwards in steps of 10, 100 and 1000 for any number up to 10 000. <i>Pupils should count on and back in steps of ten, one hundred and one thousand from different starting points. These should be practised regularly, ensuring that boundaries where more than one digit changes are included.</i> Count forwards and backwards in tenths and hundredths	 Pay particular attention where regrouping happens more than once and so more than one digit changes. e.g. $990 + 10$ or $19.9 + 0.1$
Using known facts and knowledge of place value to derive facts. Add and subtract multiples of 10, 100 and 1000 mentally <i>Pupils extend this knowledge to mentally adding and subtracting multiples of 10, 100 and 1000. Counting in different multiples of 10, 100 and 1000 should be incorporated into transition activities and practised regularly.</i>	 $2 + 4 = 6$ $20 + 40 = 60$ $200 + 400 = 600$ $2000 + 4000 = 6000$
Adding and subtracting by partitioning one number and applying known facts. <i>By Year 4 pupils are confident in their place value knowledge and are calculating mentally both with calculations that do not require regrouping and with those that do.</i>	See Y3 guidance on mental addition & subtraction, remembering that use of concrete manipulatives and images in both teaching and reasoning activities will help to secure understanding and develop mastery.

Strategies & Guidance	CPA
Round and adjust <i>Pupils should recognise that this strategy is useful when adding and subtracting near multiples of ten. They should apply their knowledge of rounding.</i> <i>It is very easy to be confused about how to adjust and so visual representations and logical reasoning are essential to success with this strategy.</i> <i>Build flexibility by completing the same calculation in a different order.</i>	$3527 + 296 = 3827 - 4$  Completing the same calculation but adjusting first: $3527 + 296 = 3523 + 300$  $4523 - 3997 = 523 + 3$  Completing the same calculation but adjusting first: $4523 - 3997 = 4526 - 4000$ 
Near doubles <i>Pupils should be able to double numbers up to 100 and use this to derive doubles for multiples of ten. These facts can be adjusted to calculate near doubles.</i>	$1600 + 1598 = \text{double } 1600 - 2$ 

Written column methods for addition

CPA

Thousands	Hundreds	Tens	Ones
			
			

Thousands	Hundreds	Tens	Ones
			
			
			

Thousands	Hundreds	Tens	Ones
			

	5	2	7	3
+	5	4	1	
	5	8	1	4

Place value counters are a useful manipulative for representing the steps of the formal written method. These should be used alongside the written layout to ensure conceptual understanding and as a tool for explaining.

Thousands	Hundreds	Tens	Ones
			
			

Thousands	Hundreds	Tens	Ones

$$\begin{array}{r} 42\cancel{3}^{15}2 \\ - 3271 \\ \hline 1081 \end{array}$$

Strategies & Guidance**Calculating with decimal numbers**

Assign different values to Dienes equipment. If a Dienes 100 block has the value of 1, then a tens rod has a value of 0.1 and a ones cube has a value of 0.01. These can then be used to build a conceptual understanding of the relationship between these.

Place value counters are another useful manipulative for representing decimal numbers.

All of the calculation strategies for integers (whole numbers) can be used to calculate with decimal numbers.

CPA $= 1$  $= 0.1$ $\text{ones cube} = 0.01$

$$24.2 + 13.4 =$$

Tens	Ones	Tenths

National Curriculum objectives linked to multiplication and division

These objectives are explicitly covered through the strategies outlined in this document:

- ☐ count from 0 in multiples of 6, 7, 9, 25 and 1000
- ☐ recall and use multiplication and division facts for multiplication tables up to 12×12
- ☐ write and calculate mathematical statements for multiplication and division using the multiplication tables that they know, including for two-digit numbers times one-digit numbers, using mental and progressing to formal written methods
- ☐ recognise and use factor pairs and commutativity in mental calculations
- ☐ use place value, known and derived facts to multiply and divide mentally, including: multiplying by 0 and 1; dividing by 1; multiplying together three numbers
- ☐ multiply two-digit and three-digit numbers by a one-digit number using formal written layout
- ☐ find the effect of dividing a one- or two-digit number by 10 and 100, identifying the value of the digits in the answer as ones, tenths and hundredths.

The following objectives should be planned for lessons where new strategies are being introduced and developed:

- ☐ solve problems involving multiplying and adding, including using the distributive law to multiply two digit numbers by one digit, integer scaling problems and harder correspondence problems such as n objects are connected to m objects.

Y4 Multiplication

Strategies & Guidance

Multiplying by 10 and 100

When you multiply by ten, each part is ten times greater. The ones become tens, the tens become hundreds, etc.

When multiplying whole numbers, a zero holds a place so that each digit has a value that is ten times greater.

Repeated multiplication by ten will build an understanding of multiplying by 100 and 1000

CPA

thousands	hundreds	tens	ones
			3
		3	0
3	0	0	0

$$3 \times 10 = 30$$

$$3 \times 100 = 300$$

$$3 \times 1000 = 3000$$

Using known facts and place value for mental multiplication involving multiples of 10 and 100

Pupils use their growing knowledge of multiplication facts, place value and derived facts to multiply mentally.

Emphasis is placed on understanding the relationship (10 times or 100 times greater) between a known number fact and one to be derived, allowing far larger 'fact families' to be derived from a single known number fact.

Knowledge of commutativity (that multiplication can be completed in any order) is used to find a range of related facts.

factor factor product
 $3 \times 7 = 21$

factor factor product
 $7 \times 3 = 21$



$$30 \times 7 = 210$$

$$300 \times 7 = 2100$$

$$70 \times 3 = 210$$

$$700 \times 3 = 2100$$

$$7 \times 30 = 210$$

$$7 \times 300 = 2100$$

$$3 \times 70 = 210$$

$$3 \times 700 = 2100$$

Strategies & Guidance

Multiplying by partitioning one

Pupils build on mental multiplication strategies and develop an explicit understanding of distributive law, which allows them to explore new strategies to make more efficient calculations.

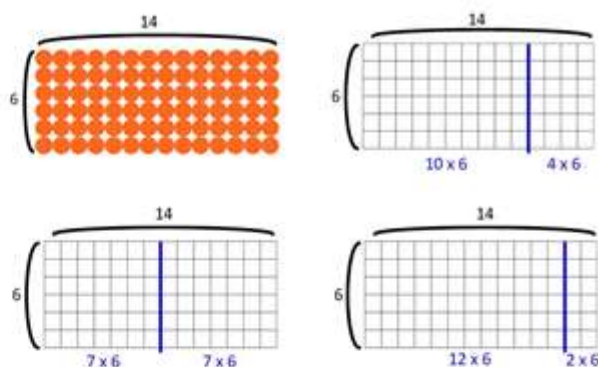
As well as partitioning into tens and ones (a familiar strategy), they begin to explore compensating strategies and factorisation to find the most efficient solution to a calculation.

Distributive law

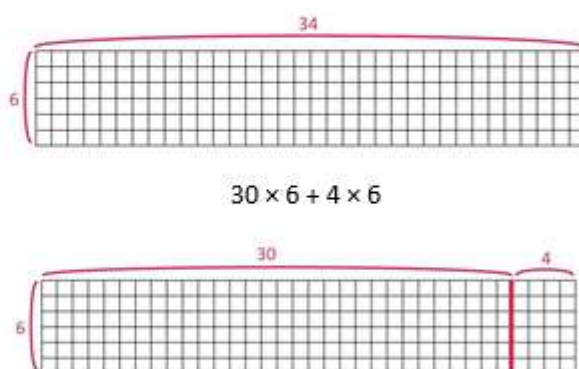
$$a \times (b + c) = a \times b + a \times c$$

CPA

$$14 \times 6$$



$$34 \times 6$$



Mental multiplication of three 1-digit numbers, using the associative law

Pupils first learn that multiplication can be performed in any order, before applying this to choose the most efficient order to complete calculations, based on their increasingly sophisticated number facts and place value knowledge.

Four pots each containing two flowers which each have seven petals. How many petals in total?



$$(4 \times 2) \times 7 \text{ or } 4 \times (2 \times 7)$$

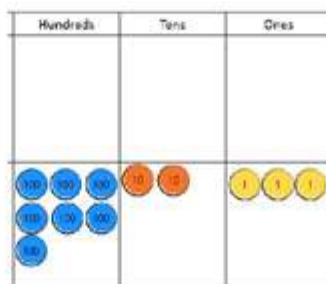
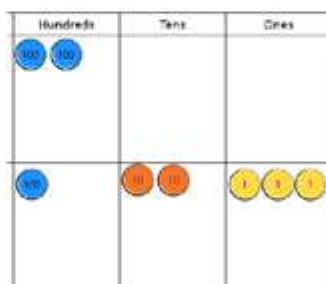
Strategies & Guidance

Short multiplication of 3-digit number by 1-digit number

To begin with pupils are presented with calculations that require no regrouping or only regrouping from the ones to the tens. Their conceptual understanding is supported by the use of place value counters, both during teacher demonstrations and during their own practice.

With practice pupils will be able to regroup in any column, including from the hundreds to the thousands, including being able to multiply numbers containing zero and regrouping through multiple columns in a single calculation.

CPA



To calculate 241×3 , represent the number 241. Multiply each part by 3, regrouping as needed.

$$\begin{array}{r}
 241 \\
 \times \quad 3 \\
 \hline
 723 \\
 1
 \end{array}$$

Y4 Division

Strategies & Guidance

Dividing by 10 and 100

When you divide by ten, each part is ten times smaller. The hundreds become tens and the tens become ones. Each digit is in a place that gives it a value that is ten times smaller.

When dividing multiples of ten, a place holder is no longer needed so that each digit has a value that is ten times smaller. E.g. $210 \div 10 = 21$

CPA

thousands	hundreds	tens	ones
			
			
		3	0
		0	0
3	0	0	0

$$30 \div 10 = 3$$

$$300 \div 100 = 3$$

$$3000 \div 1000 = 3$$

$$300 \div 10 = 30$$

$$3000 \div 100 = 30$$

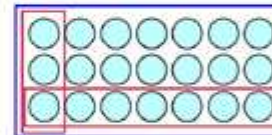
$$3000 \div 10 = 300$$

Derived facts

Pupils use their growing knowledge of multiplication facts, place value and derived facts to multiply mentally.

Understanding of the inverse relationship between multiplication and division allows corresponding division facts to be derived.

$$21 \div 3 = 7$$



$$21 \div 7 = 3$$



$$210 \div 7 = 30$$

$$2100 \div 7 = 300$$

$$210 \div 3 = 70$$

$$2100 \div 3 = 700$$

$$210 \div 30 = 7$$

$$2100 \div 300 = 7$$

$$210 \div 70 = 3$$

$$2100 \div 700 = 3$$

Strategies & Guidance

Short division of 4-digit numbers by 1-digit numbers

Pupils start with dividing 4-digit numbers by 2, 3 and 4, where no regrouping is required. Place value counters are used simultaneously in a place value chart, to develop conceptual understanding.

They progress to calculations that require regrouping in the hundreds or tens columns.

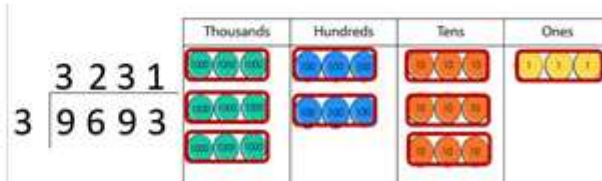
Pupils build on their conceptual knowledge of division to become confident with dividing numbers where the tens digit is smaller than the divisor, extending this to any digit being smaller than the divisor.

CPA

Division as sharing



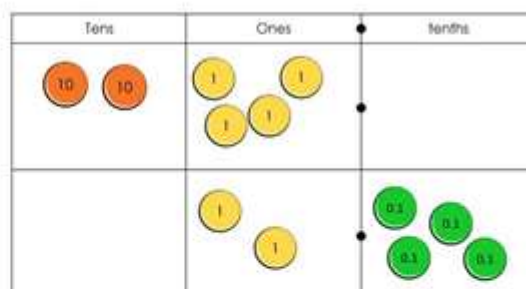
Division as grouping



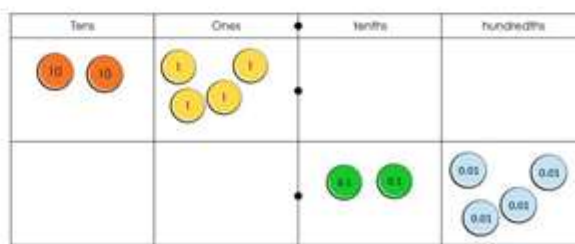
Division of a one- or two-digit number by 10 and 100, identifying the value of the digits in the answer as ones, tenths and hundredths

When you divide by ten, each part is ten times smaller. The tens become ones and the ones become tenths. Each digit is in a place that gives it a value that is ten times smaller.

$$24 \div 10 = 2.4$$



$$24 \div 100 = 0.24$$



Progression in calculations

Year 5 + Year 6

Year 5 and Year 6 are together because the calculation strategies used are broadly similar, with Year 6 using larger and smaller numbers. Any differences for Year 6 are highlighted in red.

National Curriculum objectives linked to integer addition and subtraction

These objectives are explicitly covered through the strategies outlined in this document:

- ☐ add and subtract numbers mentally with increasingly large numbers
- ☐ add and subtract whole numbers with more than 4 digits, including using formal written methods (columnar addition and subtraction)
- ☐ use negative numbers in context, and calculate intervals across zero
- ☐ perform mental calculations, including with mixed operations and large numbers
- ☐ use estimation to check answers to calculations and determine, in the context of a problem, an appropriate degree of accuracy

The following objectives should be planned for lessons where new strategies are being introduced and developed:

- ☐ use rounding to check answers to calculations and determine, in the context of a problem, levels of accuracy
- ☐ solve addition and subtraction multi-step problems in contexts, deciding which operations and methods to use and why
- ☐ solve problems involving addition, subtraction, multiplication and division and a combination of these, including understanding the meaning of the equals sign.

Strategies & Guidance

Count forwards or backwards in steps of

powers of 10 for any given number up to 1 000 000

Skip counting forwards and backwards in steps of powers of 10 (i.e. 10, 100, 1000, 10 000 and 100 000) should be incorporated into transition activities and practised regularly.

In Year 5 pupils work with numbers up to 1 000 000 as well as tenths, hundredths and thousandths.

In Year 6 pupils work with numbers up to 10 000 000.

CPA

Support with place value counters on a place value chart, repeatedly adding the same counter and regrouping as needed.

Hundred thousands	Ten thousands	Thousands	Hundreds	Tens	Ones	Tenths	Hundredths	Thousandths



Using known facts and understanding of place value to derive

Using the following language makes the logic explicit: I know three ones plus four ones is equal to seven ones. Therefore, three ten thousands plus four ten thousands is equal to seven ten thousands.

In Year 5 extend to multiples of 10 000 and 100 000 as well as tenths, hundredths and thousandths.

In Year 6 extend to multiples of one million.

These derived facts should be used to estimate and check answers to calculations.

3 + 4 = 7

30 000 + 40 000 = 70 000

300 000 + 400 000 = 700 000

20 000 + 40 000 = 60 000

40 000 + 20 000 = 60 000

60 000 - 40 000 = 20 000

60 000 - 20 000 = 40 000

0.6 = 0.2 + 0.4

0.6 = 0.4 + 0.2

0.2 = 0.6 - 0.4

0.4 = 0.6 - 0.2

CPA

Partitioning one number and applying known facts to add.

Pupils can use this strategy mentally or with jottings as

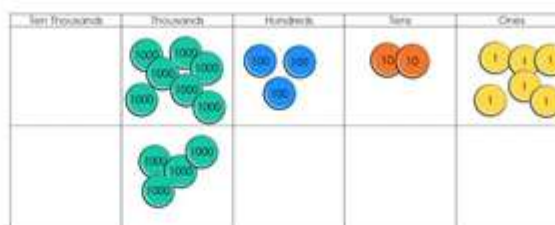
Pupils should be aware of the range of choices available when deciding how to partition the number that is to be added.

They should be encouraged to count on from the number of greater value as this will be more efficient. However, they should have an understanding of the commutative law of addition, that the parts can be added in any order.

Pupils have experience with these strategies with smaller numbers from previous years and so the focus should be on developing flexibility and exploring efficiency.

Partitioning into place value amounts (canonical partitioning):

$$4650 + 7326 = 7326 + 4000 + 600 + 50$$



Partitioning in different ways (non-canonical partitioning):

Extend the 'Make ten' strategy (see guidance in Y1 or Y2) to

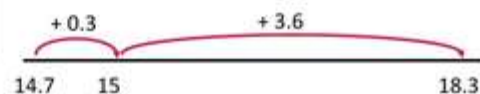
count on to a multiple of 10.

$$6785 + 2325 = 6785 + 15 + 200 + 2110$$



The strategy can be used with decimal numbers, Make one:

$$14.7 + 3.6 = 14.7 + 0.3 + 3.3 = 15 + 3.3$$



Subtraction by partitioning and applying known facts.

Pupils can use this strategy mentally or with jottings as needed.

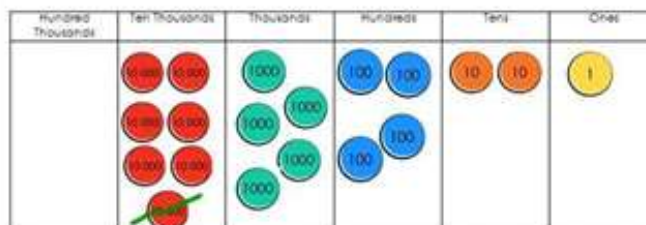
range of choices available when deciding how to partition the number that is to be subtracted.

Pupils have experience with these strategies with smaller numbers from previous years and so the focus should be on developing flexibility and exploring efficiency.

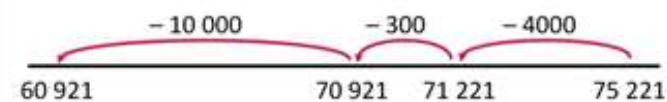
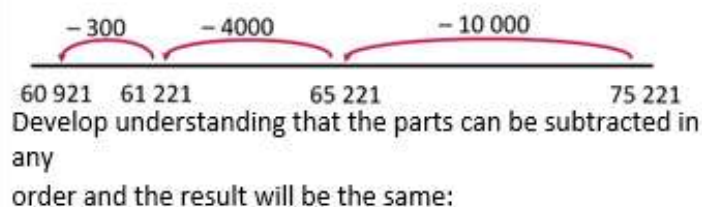
CPA

Partitioning into place value amounts (canonical partitioning):

$$75\ 221 - 14\ 300 = 75\ 221 - 10\ 000 - 4000 - 300$$

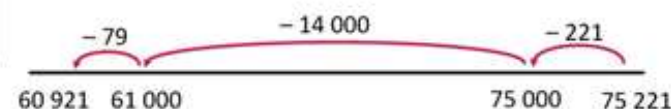


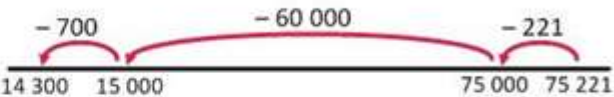
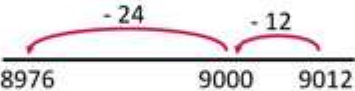
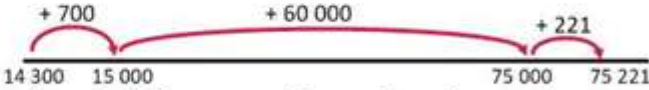
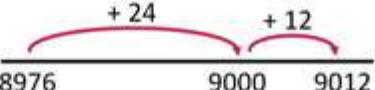
Represent pictorially with a number line, starting on the right and having the arrows jump to the left:

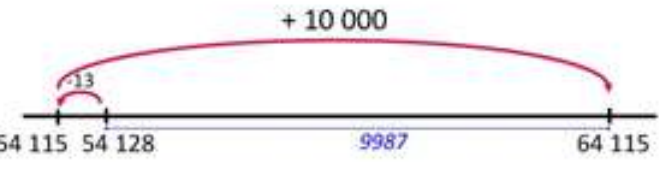
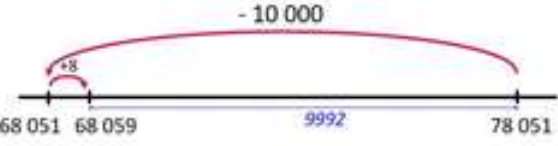
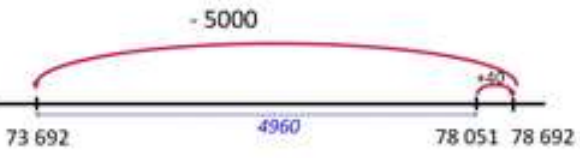


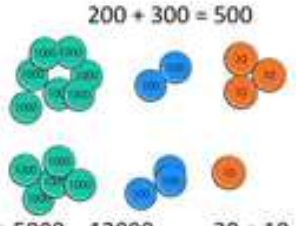
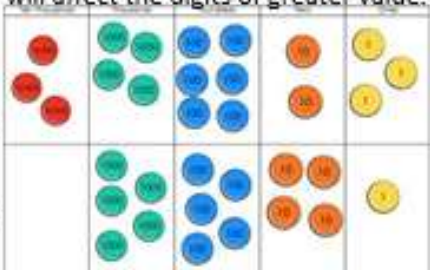
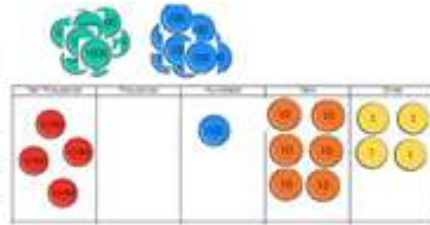
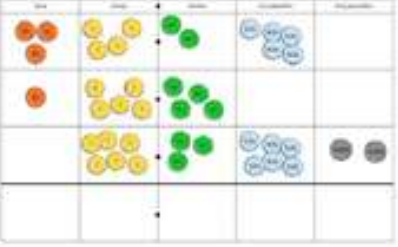
Partitioning in different ways (non-canonical partitioning):

Extend the 'Make ten' strategy (see guidance in Y1 or Y2) to count back to a multiple of 10.



Strategies & Guidance	CPA
Calculate difference by “counting back” <i>It is interesting to note that finding the difference is reversible. For example, the difference between 5 and 2 is the same as the difference between 2 and 5. This is not the case for other subtraction concepts.</i>	$75\,221 - 14\,300$ Place the numbers either end of a number line and work out the difference between them. Select efficient jumps.  Finding the difference is efficient when the numbers are close to each other: $9012 - 8976$ 
Calculate difference by “counting on” <i>Addition strategies can be used to find difference.</i>	$75\,221 - 14\,300$  Finding the difference is efficient when the numbers are close to each other $9012 - 8976$ 

Strategies & Guidance	CPA
Round and adjust Addition and subtraction <i>Pupils should recognise that this strategy is useful when adding and subtracting near multiples of ten. They should apply their knowledge of rounding.</i> <i>It is very easy to be confused about how to adjust and so visual representations and logical reasoning are essential to success with this strategy.</i>	Addition  54 128 64 115 64 128 $54\,128 + 9987 = 54\,128 + 10\,000 - 13 = 64\,128 - 13$ Pupils should realise that they can adjust first:  54 115 54 128 64 115 $54\,128 + 9987 = 54\,128 - 13 + 10\,000 = 54\,115 + 10\,000$ Subtraction  68 051 68 059 78 051 $78\,051 - 9992 = 78\,051 - 10\,000 + 8 = 68\,051 + 8$ Pupils should realise that they can adjust first:  73 692 78 051 78 692 $78\,051 - 4960 = 78\,051 + 40 - 5000 = 78\,692 - 5000$
Near doubles <i>Pupils should be able to double numbers up to 100 and use this to derive doubles for multiples of ten as well as decimal numbers. These facts can be adjusted to calculate near doubles.</i>	16 $0 + 170 = \text{double } 150 + 10 + 20$ 16 $0 + 170 = \text{double } 160 + 10 \text{ or}$ 16 $0 + 170 = \text{double } 170 - 10$ $2.5 + 2.6 = \text{double } 2.5 + 0.1$

Strategies & Guidance	CPA
Partition both numbers and combine the parts <i>method for numbers up to 10 000, using place value counters or Dienes to show conceptual understanding.</i> <i>If multiple regroupings are required, then pupils should consider using the column method.</i>	$7230 + 5310 = 12\ 000 + 500 + 40$ $200 + 300 = 500$  $7000 + 5000 = 12000$ $30 + 10 = 40$ Pupils should be aware that the parts can be added in any order.
Written column methods for addition <i>In Year 5, pupils are expected to be able to use formal written methods to add whole numbers with more than four digits as well as working with numbers with up to three decimal places.</i> <i>Pupils should think about whether this is the most efficient method, considering if mental methods would be more effective.</i> <i>Continue to use concrete manipulatives alongside the formal method.</i> <i>When adding decimal numbers with a different number of decimal places, in order to avoid calculation errors, pupils should be encouraged to insert zeros so that there is a digit in every row. This is not necessary for calculation and these zeros are not place holders as the value of the other digits is not changed by it being placed.</i>	For this method start with the digit of least value because if regrouping happens it will affect the digits of greater value.  Combine the counters in each column and regroup as needed:  Decimal numbers:  $\begin{array}{r} 3\ 4\ 6\ 2\ 3 \\ +\ 5\ 5\ 4\ 1 \\ \hline 1\ 0\ 1\ 6\ 4 \end{array}$ $\begin{array}{r} 3\ 4\ .\ 2\ 5 \\ 1\ 5\ .\ 4 \\ +\ 6\ .\ 3\ 6\ 2 \\ \hline 5\ 6\ .\ 0\ 1\ 2 \end{array}$

Strategies & Guidance	CPA
Written column methods for subtraction <i>In Year 5, pupils are expected to be able to use formal written methods to subtract whole numbers with more than four digits as well as working with numbers with up to three decimal places.</i> <i>Pupils should be given plenty of practice with calculations that require multiple separate instances of regrouping.</i> <i>In Year 3 and 4 they become more familiar with calculations that require 'regrouping to regroup'. Understanding must be secured through the considered use of manipulatives and images, combined with careful use of language.</i> <i>Pupils should think about if this is the most efficient method, considering whether mental strategies (such as counting on, using known number facts, compensation etc.) may be likelier to produce an accurate solution.</i>	$\begin{array}{r} 41362 \\ - 32243 \\ \hline \end{array}$ $\begin{array}{r} 3151 \\ 41362 \\ - 32243 \\ \hline 9118 \end{array}$ The term regrouping should be the language used. You can use the terms 'exchange' with subtraction but it needs careful consideration. You can regroup 62 as 50 and 12 (5 tens and 12 ones) instead of 60 and 2 (6 tens and 12 ones). Or you can 'exchange' one of the tens for 10 ones resulting in 5 tens and 12 ones. If you have exchanged, then the number has been regrouped.

Progression in calculations

Year 5 + Year 6

National Curriculum objectives linked to multiplication and division

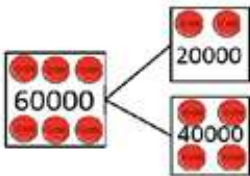
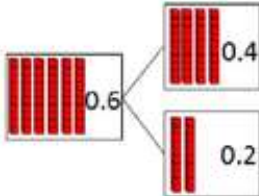
These objectives are explicitly covered through the strategies outlined in this document:

- ☐ multiply and divide whole numbers by 10, 100 and 1000
- ☐ multiply numbers up to 4 digits by a one- or two-digit number using a formal written method, including long multiplication for two-digit numbers
- ☐ multiply and divide numbers mentally drawing upon known facts
- ☐ divide numbers up to 4 digits by a one-digit number using the formal written method of short division and interpret remainders appropriately for the context
- ☐ multiply multi-digit numbers up to 4 digits by a two-digit whole number using the formal written method of long multiplication
- ☐ divide numbers up to 4 digits by a two-digit whole number using the formal written method of long division, and interpret remainders as whole number remainders, fractions, or by rounding, as appropriate for the context
- ☐ divide numbers up to 4 digits by a two-digit number using the formal written method of short division where appropriate, interpreting remainders according to the context
- ☐ multiply one-digit numbers with up to two decimal places by whole numbers
- ☐ use written division methods in cases where the answer has up to two decimal places

The following objectives should be planned for lessons where new strategies are being introduced and developed:

- ☐ solve problems involving multiplication and division including using their knowledge of factors and multiples, squares and cubes
- ☐ solve problems involving addition, subtraction, multiplication and division and a combination of these, including understanding the meaning of the equals sign
- ☐ use their knowledge of the order of operations to carry out calculations involving the four operations
- ☐ solve addition and subtraction multi-step problems in contexts, deciding which operations and methods to use and why
- ☐ solve problems involving addition, subtraction, multiplication and division
- ☐ solve problems involving the relative sizes of two quantities where missing values can be found by using integer multiplication and division facts.

Y5 and Y6 Addition & Subtraction

Strategies & Guidance	CPA																		
Count forwards or backwards in steps of powers of 10 for any given number up to 1 000 000 <i>Skip counting forwards and backwards in steps of powers of 10 (i.e. 10, 100, 1000, 10 000 and 100 000) should be incorporated into transition activities and practised regularly.</i> <i>In Year 5 pupils work with numbers up to 1 000 000 as well as tenths, hundredths and thousandths.</i> <i>In Year 6 pupils work with numbers up to 10 000 000.</i>	Support with place value counters on a place value chart, repeatedly adding the same counter and regrouping as needed. <table border="1"><thead><tr><th>Hundred Thousands</th><th>Ten Thousands</th><th>Thousands</th><th>Hundreds</th><th>Tens</th><th>Ones</th><th>Tenths</th><th>Hundredths</th><th>Thousandths</th></tr></thead><tbody><tr><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td></tr></tbody></table> Counting sticks & number lines  Pay attention to when exchanging happens more than once and so more than one digit changes. e.g. $9900 + 100 = 10\ 000$ or $99\ 000 + 1000 = 100\ 000$	Hundred Thousands	Ten Thousands	Thousands	Hundreds	Tens	Ones	Tenths	Hundredths	Thousandths									
Hundred Thousands	Ten Thousands	Thousands	Hundreds	Tens	Ones	Tenths	Hundredths	Thousandths											
Using known facts and understanding of place value to derive <i>Using the following language makes the logic explicit: I know three ones plus four ones is equal to seven ones. Therefore, three ten thousands plus four ten thousands is equal to seven ten thousands.</i> <i>In Year 5 extend to multiples of 10 000 and 100 000 as well as tenths, hundredths and thousandths.</i> <i>In Year 6 extend to multiples of one million.</i> <i>These derived facts should be used to estimate and check answers to calculations.</i>	 $3 + 4 = 7$  $30\ 000 + 40\ 000 = 70\ 000$  $300\ 000 + 400\ 000 = 700\ 000$ $20\ 000 + 40\ 000 = 60\ 000$ $40\ 000 + 20\ 000 = 60\ 000$ $60\ 000 - 40\ 000 = 20\ 000$ $60\ 000 - 20\ 000 = 40\ 000$   $0.6 = 0.2 + 0.4$ $0.6 = 0.4 + 0.2$ $0.2 = 0.6 - 0.4$ $0.4 = 0.6 - 0.2$																		

CPA

Partitioning one number and applying known facts to add.

Pupils can use this strategy mentally or with jottings as

Pupils should be aware of the range of choices available when deciding how to partition the number that is to be added.

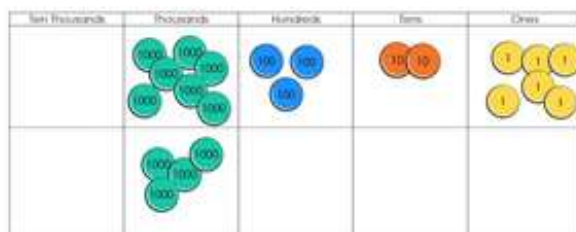
They should be encouraged to count on from the number of greater value as this will be

more efficient. However, they should have an understanding of the commutative law of addition, that the parts can be added in any order.

Pupils have experience with these strategies with smaller numbers from previous years and so the focus should be on developing flexibility and exploring efficiency.

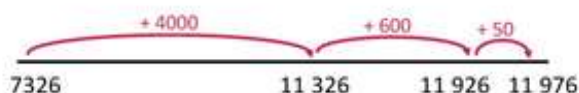
Partitioning into place value amounts (canonical partitioning):

$$4650 + 7326 = 7326 + 4000 + 600 + 50$$



The representation above shows the thousands being added.

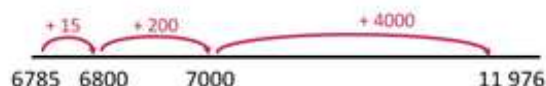
Represent pictorially with an empty number line:



Partitioning in different ways (non-canonical partitioning):

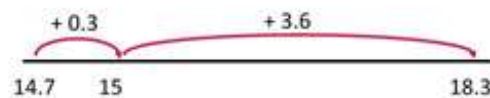
Extend the 'Make ten' strategy (see guidance in Y1 or Y2) to count on to a multiple of 10.

$$6785 + 2325 = 6785 + 15 + 200 + 2110$$



The strategy can be used with decimal numbers, Make one:

$$14.7 + 3.6 = 14.7 + 0.3 + 3.3 = 15 + 3.3$$



CPA

Subtraction by partitioning and applying known facts.

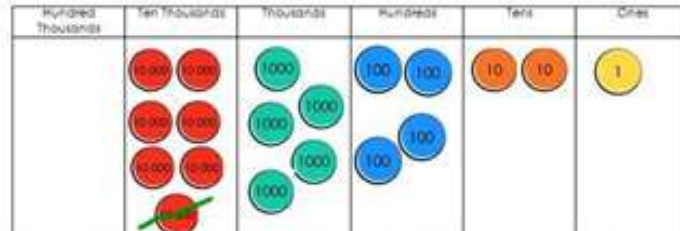
Pupils can use this strategy mentally or with jottings as needed.

range of choices available when deciding how to partition the number that is to be subtracted.

Pupils have experience with these strategies with smaller numbers from previous years and so the focus should be on developing flexibility and exploring efficiency.

Partitioning into place value amounts (canonical partitioning):

$$75\,221 - 14\,300 = 75\,221 - 10\,000 - 4\,000 - 300$$

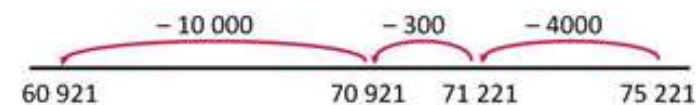


Represent pictorially with a number line, starting on the right and having the arrows jump to the left:



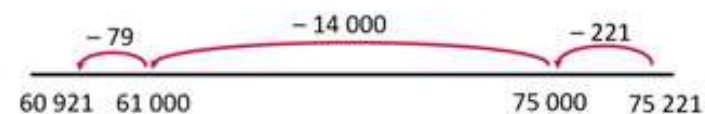
Develop understanding that the parts can be subtracted in any

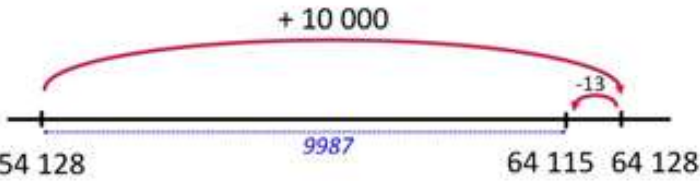
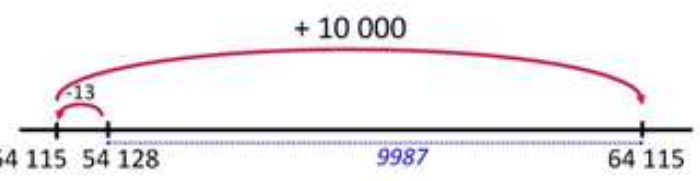
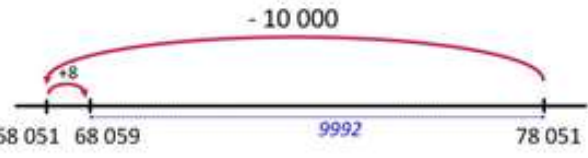
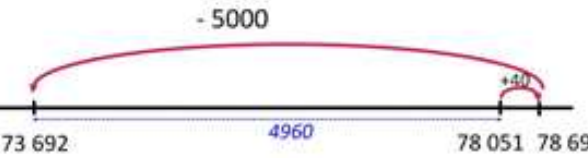
order and the result will be the same:

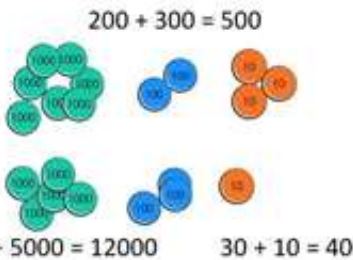
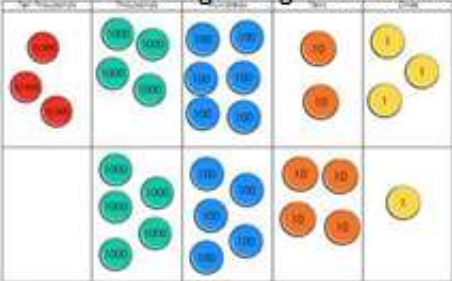
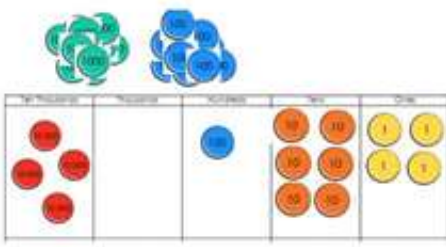
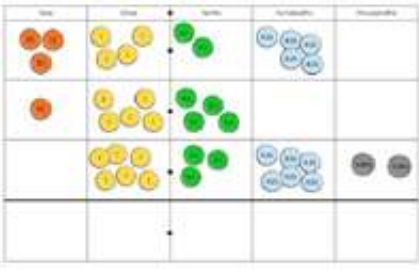


Partitioning in different ways (non-canonical partitioning):

Extend the 'Make ten' strategy (see guidance in Y1 or Y2) to count back to a multiple of 10.



Strategies & Guidance	CPA
Round and adjust Addition and subtraction <i>Pupils should recognise that this strategy is useful when adding and subtracting near multiples of ten. They should apply their knowledge of rounding.</i> <i>It is very easy to be confused about how to adjust and so visual representations and logical reasoning are essential to success with this strategy.</i>	Addition  $54\ 128 + 9987 = 54\ 128 + 10\ 000 - 13 = 64\ 128 - 13$ Pupils should realise that they can adjust first:  $54\ 128 + 9987 = 54\ 128 - 13 + 10\ 000 = 54\ 115 + 10\ 000$ Subtraction  $78\ 051 - 9992 = 78\ 051 - 10\ 000 + 8 = 68\ 051 + 8$ Pupils should realise that they can adjust first:  $78\ 051 - 4960 = 78\ 051 + 40 - 5000 = 78\ 692 - 5000$
Near doubles <i>Pupils should be able to double numbers up to 100 and use this to derive doubles for multiples of ten as well as decimal numbers. These facts can be adjusted to calculate near doubles.</i>	16 $0 + 170 = \text{double } 150 + 10 + 20$ 16 $0 + 170 = \text{double } 160 + 10 \text{ or}$ 16 $0 + 170 = \text{double } 170 - 10$ $2.5 + 2.6 = \text{double } 2.5 + 0.1$

Strategies & Guidance	CPA
Partition both numbers and combine the parts <i>method for numbers up to 10 000, using place value counters or Dienes to show conceptual understanding.</i> <i>If multiple regroupings are required, then pupils should consider using the column method.</i>	$7230 + 5310 = 12\ 000 + 500 + 40$  Pupils should be aware that the parts can be added in any order.
Written column methods for addition <i>In Year 5, pupils are expected to be able to use formal written methods to add whole numbers with more than four digits as well as working with numbers with up to three decimal places.</i> <i>Pupils should think about whether this is the most efficient method, considering if mental methods would be more effective.</i> <i>Continue to use concrete manipulatives alongside the formal method.</i> <i>When adding decimal numbers with a different number of decimal places, in order to avoid calculation errors, pupils should be encouraged to insert zeros so that there is a digit in every row. This is not necessary for calculation and these zeros are not place holders as the value of the other digits is not changed by it being placed.</i>	For this method start with the digit of least value because if regrouping happens it will affect the digits of greater value.  Combine the counters in each column and regroup as needed:  Decimal numbers: 

Strategies & Guidance	CPA
Written column methods for subtraction <i>In Year 5, pupils are expected to be able to use formal written methods to subtract whole numbers with more than four digits as well as working with numbers with up to three decimal places.</i> <i>Pupils should be given plenty of practice with calculations that require multiple separate instances of regrouping.</i> <i>In Year 3 and 4 they become more familiar with calculations that require 'regrouping to regroup'. Understanding must be secured through the considered use of manipulatives and images, combined with careful use of language.</i> <i>Pupils should think about if this is the most efficient method, considering whether mental strategies (such as counting on, using known number facts, compensation etc.) may be likelier to produce an accurate solution.</i>	$\begin{array}{r} 41362 \\ - 32243 \\ \hline \end{array}$ $\begin{array}{r} 3 1 5 1 \\ 41362 \\ - 32243 \\ \hline 9118 \end{array}$ The term regrouping should be the language used. You can use the terms 'exchange' with subtraction but it needs careful consideration. You can regroup 62 as 50 and 12 (5 tens and 12 ones) instead of 60 and 2 (6 tens and 12 ones). Or you can 'exchange' one of the tens for 10 ones resulting in 5 tens and 12 ones. If you have exchanged, then the number has been regrouped.

Progression in calculations

Year 5 + Year 6

National Curriculum objectives linked to multiplication and division

These objectives are explicitly covered through the strategies outlined in this document:

- multiply and divide whole numbers by 10, 100 and 1000
- multiply numbers up to 4 digits by a one- or two-digit number using a formal written method, including long multiplication for two-digit numbers
- multiply and divide numbers mentally drawing upon known facts
- divide numbers up to 4 digits by a one-digit number using the formal written method of short division and interpret remainders appropriately for the context
- multiply multi-digit numbers up to 4 digits by a two-digit whole number using the formal written method of long multiplication
- divide numbers up to 4 digits by a two-digit whole number using the formal written method of long division, and interpret remainders as whole number remainders, fractions, or by rounding, as appropriate for the context
- divide numbers up to 4 digits by a two-digit number using the formal written method of short division where appropriate, interpreting remainders according to the context
- multiply one-digit numbers with up to two decimal places by whole numbers
- use written division methods in cases where the answer has up to two decimal places

The following objectives should be planned for lessons where new strategies are being introduced and developed:

- solve problems involving multiplication and division including using their knowledge of factors and multiples, squares and cubes
- solve problems involving addition, subtraction, multiplication and division and a combination of these, including understanding the meaning of the equals sign
- use their knowledge of the order of operations to carry out calculations involving the four operations
- solve addition and subtraction multi-step problems in contexts, deciding which operations and methods to use and why
- solve problems involving addition, subtraction, multiplication and division
- solve problems involving the relative sizes of two quantities where missing values can be found by using integer multiplication and division facts.

Y5 and Y6 Multiplication

Strategies & Guidance

Multiply and divide whole numbers and those involving decimals by 10, 100 and 1000

Avoid saying that you "add a zero" when multiplying by ten and instead use the language of place holder.

Use place value counters and charts to visualise and then notice what happens to the digits.

CPA

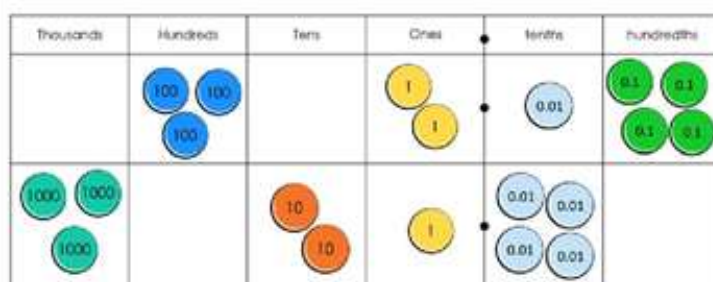
When you multiply by ten, each part is ten times greater. The ones

become tens, the tens become hundreds, etc.

When multiplying whole numbers, a zero holds a place so that each

digit has a value that is ten times greater.

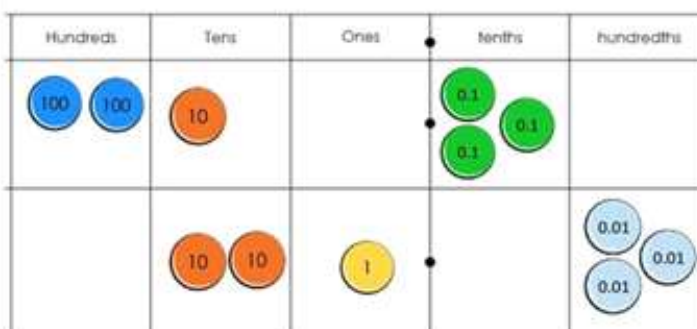
$$102.14 \times 10 = 1021.4$$



When you divide by ten, each part is ten times smaller. The hundreds become tens and the tens become ones. Each digit is in a place that gives it a value that is ten times smaller.

When dividing multiples of ten, a place holder is no longer needed so that each digit has a value that is ten times smaller. E.g. $210 \div 10 = 21$

$$210.3 \div 10 = 21.03$$



Strategies & Guidance

Using known facts and place value to derive multiplication facts

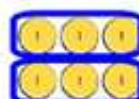
Emphasis is placed on understanding the relationship (10 times or 100 times greater) between a known number fact and one to be derived, allowing far larger 'fact families' to be derived from a single known number fact.

Knowledge of commutativity is further extended and applied to find a range of related facts.

Pupils should work with decimals with up to two decimal places.

These derived facts should be used to estimate and check answers to calculations.

CPA



$$2 \times 3 = 6$$

$$3 \times 2 = 6$$

$$2 \times 30 = 60$$

$$30 \times 2 = 60$$



$$2 \times 3 \times 10 = 60$$

$$3 \times 20 = 60$$

$$20 \times 3 = 60$$

$$2 \times 300 = 600$$

$$300 \times 2 = 600$$

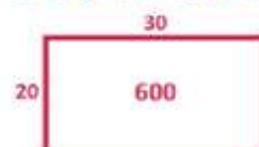


$$2 \times 30 \times 10 = 600$$

$$20 \times 3 \times 10 = 600$$

$$3 \times 200 = 600$$

$$200 \times 3 = 600$$



$$20 \times 30 = 600$$

$$30 \times 20 = 600$$

These are the multiplication facts pupils should be able to derive from a known fact

2 100 000	700 000 x 3	70 000 x 30	7000 x 300	700 x 3000	70 x 30 000	7 x 300 000
210 000	70 000 x 3	7000 x 30	700 x 300	70 x 3000	7 x 30 000	
21 000	7000 x 3	700 x 30	70 x 300	7 x 3000		
2100	700 x 3	70 x 30	7 x 300			
210	70 x 3	7 x 30				
21	= 7 x 3					
2.1	0.7 x 3	7 x 0.3				
0.21	0.07 x 3	0.7 x 0.3	7 x 0.03			
0.021	0.007 x 3	0.07 x 0.3	0.7 x 0.03	7 x 0.003		

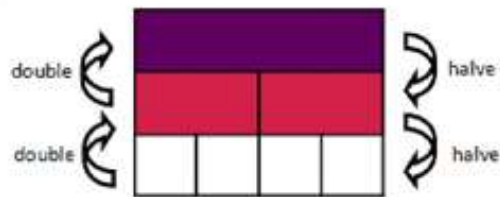
Strategies & Guidance

Doubling and halving

Pupils should experience doubling and halving larger and smaller numbers as they expand their understanding of the number system.

Doubling and halving can then be used in larger calculations.

CPA

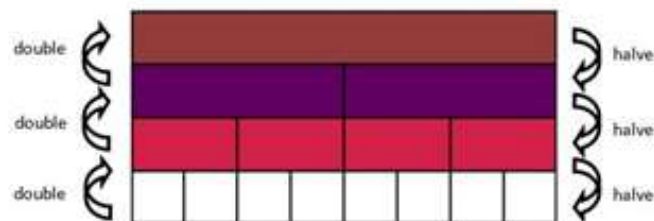


Multiply by 4 by doubling and doubling again

e.g. $16 \times 4 = 32 \times 2 = 64$

Divide by 4 by halving and halving again

e.g. $104 \div 4 = 52 \div 2 = 26$

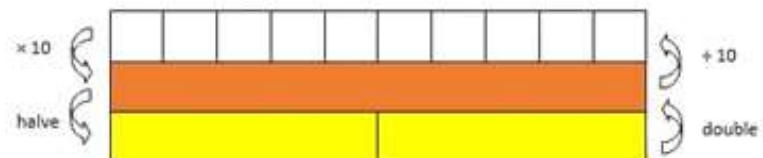


Multiply by 8 by doubling three times

e.g. $12 \times 8 = 24 \times 4 = 48 \times 2 = 96$

Divide by 8 by halving three times

e.g. $104 \div 8 = 52 \div 4 = 26 \div 2 = 13$

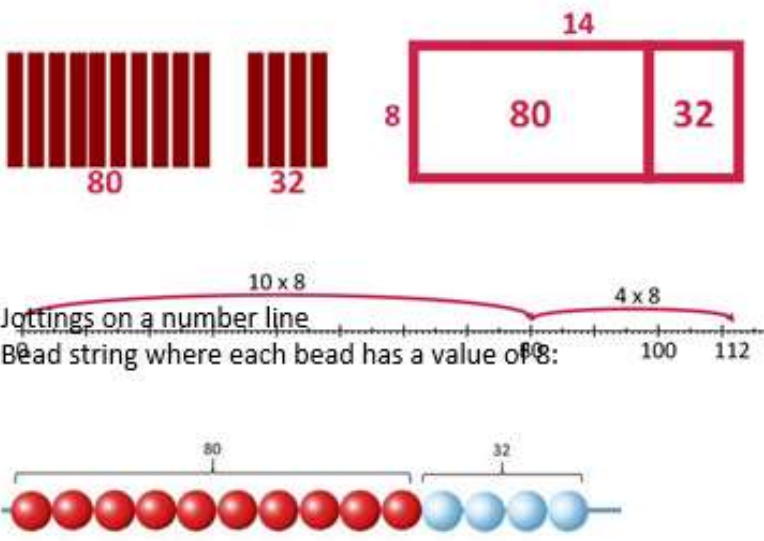
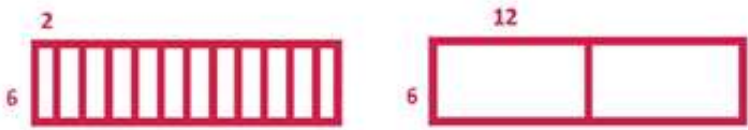


Multiply by 5 by multiplying by 10 then halving,

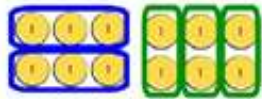
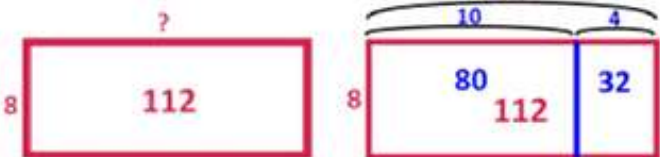
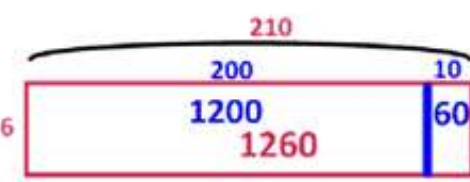
e.g. $18 \times 5 = 180 \div 2 = 90$.

Divide by 5 by dividing by 10 and doubling,

e.g. $460 \div 5 = \text{double } 46 = 92$

Strategies & Guidance	CPA
Multiply by partitioning one number and multiplying each part Distributive law $a \times (b + c) = a \times b + a \times c$ Build on pupils' understanding of arrays of counters to represent multiplication to see that area models can be a useful representation:	$8 \times 14 = 8 \times 10 + 8 \times 4$  Jottings on a number line Bead string where each bead has a value of 8:
Using knowledge of factors <i>In Year 5 pupils are expected to be able to identify factor pairs and this knowledge can be used to calculate.</i> <i>Pupils will be using the commutative and associative laws of multiplication.</i> Commutative law $a \times b = b \times a$ Associative law $a \times b \times c = (a \times b) \times c$ $= a \times (b \times c)$ <i>They should explore and compare the different options and choose the most efficient order to complete calculations.</i>	Calculate 6×24 by using factor pairs of 24 Two and twelve are factors of 24: $6 \times 2 \times 12$ $6 \times 12 \times 2$  Three and eight are factors of 24: $6 \times 3 \times 8$ $6 \times 8 \times 3$  Four and six are factors of 24: $6 \times 4 \times 6$ $6 \times 6 \times 4$ 

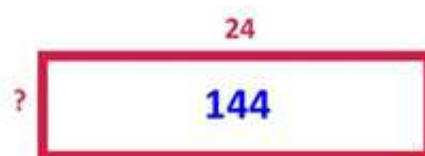
Y5 and Y6 Division

Strategies & Guidance	CPA
Deriving facts from known facts <i>Pupils use their growing knowledge of multiplication facts, place value and derived facts to multiply mentally.</i> <i>Understanding of the inverse relationship between multiplication and division allows corresponding division facts to be derived.</i>	 $6 \div 2 = 3 \qquad 6 \div 3 = 2$  $60 \div 2 = 30 \qquad 60 \div 30 = 2$  $600 \div 2 = 300 \qquad 600 \div 300 = 2$
Using knowledge of multiples to divide <i>Using an area model to partition the whole into multiples of the divisor (the number you are dividing by).</i>	 $112 \div 8 = 80 \div 8 + 32 \div 8$  $1260 \div 6 = 1200 \div 6 + 60 \div 6$ How many equal parts?  

Strategies & Guidance**Using knowledge of factors to divide**

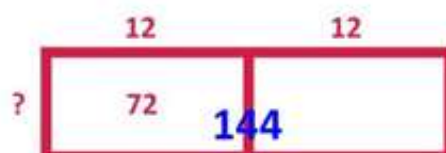
Pupils explore this strategy when using repeated halving.

$2 \times 2 = 4$ and so if you divide by 4 the same result can be achieved by dividing by two and then by two again.

CPA

$$144 \div 24$$

I know 2 and 12 are a factor pair of 24 and so I can divide by 2 and then by 12.



$$144 \div 2 \div 12$$

Strategies & Guidance

Short division

Dividing a 4-digit numbers by 1-digit numbers

The thought process of the traditional algorithm is as follows:

How many 4s in 8? 2

How many 4s in 5? 1 with 1 remaining so regroup.

How many 4s in 12? 3

How many 4s in 8? 2

Warning: If you simply apply place value knowledge to each step, the thinking goes wrong if you have to regroup.

How many 4s in 8000? 2000

How many 4s in 500?

100 with 1 remaining (illogical)

The answer would be 125.

Sharing the dividend builds conceptual understanding however doesn't scaffold the "thinking" of the algorithm.

Using place value counters and finding groups of the divisor for each power of ten will build conceptual understanding of the short division algorithm.

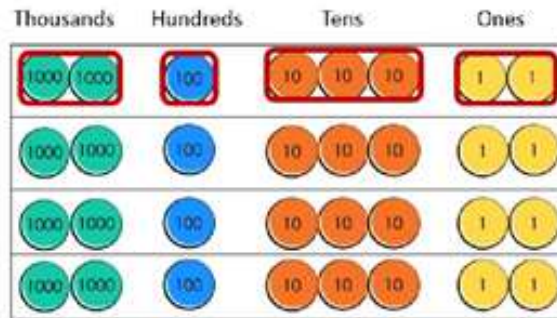
Area models are also useful representations, as seen with other strategies and exemplified for long division.

CPA

$$8528 \div 4$$

Sharing

$$\begin{array}{r} 2132 \\ 4 \overline{) 8528} \end{array}$$



8 thousands shared into 4 equal groups

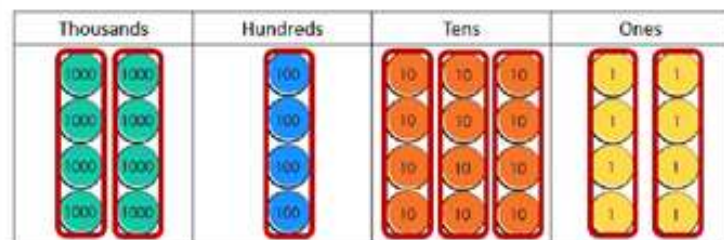
5 hundreds shared into 4 equal groups

Regroup 1 hundred for 10 tens

12 tens shared into 4 equal groups

8 ones shared into 4 equal groups.

Grouping



How many groups of 4 thousands in 8 thousands?

How many groups of 4 hundreds in 5 hundreds?

Regroup 1 hundred for 10 tens.

How many groups of 4 tens in 12 tens?

How many groups of 4 ones in 8 ones?

Strategies & Guidance**Long division****Dividing a 4-digit number by a 2-digit number**

Follow the language structures of the short division strategy. Instead of recording the regrouped amounts as small digits the numbers are written out below. This can be easier to work with when dividing by larger numbers.

If dividing by a number outside of their known facts, pupils should start by recording some multiples of that number to scaffold.

CPA

$$\begin{array}{r} 34 \\ 12 \overline{) 408} \\ \underline{36} \\ 48 \\ \underline{48} \\ 0 \end{array}$$

$$408 \div 12$$

